

EXERCISE 29.3

Evaluate the following limits:

$$1. \lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5}$$

Solution:

Given:
The limit $\lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5}$

By substituting the value of x, we get

$$\begin{aligned} \lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5} &= \frac{2(-5)^2 + 9(-5) - 5}{(-5) + 5} \\ &= \frac{50 - 50}{(-5) + 5} \\ &= \frac{0}{0} \end{aligned}$$

[Since, it is of the form indeterminate]

By using factorization method:

$$\begin{aligned} \lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5} &= \lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x + 5} \\ &= \lim_{x \rightarrow -5} \frac{2x^2 + 10x - x - 5}{x + 5} \\ &= \lim_{x \rightarrow -5} \frac{2x(x + 5) - (x + 5)}{x + 5} \\ &= \lim_{x \rightarrow -5} \frac{(2x - 1)(x + 5)}{x + 5} \\ &= \lim_{x \rightarrow -5} 2x - 1 \\ &= 2(-5) - 1 \\ &= -11 \end{aligned}$$

∴ The value of the given limit is – 11.

$$2. \lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$$

Solution:

Given:
The limit $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$

By substituting the value of x, we get

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} &= \frac{(3)^2 - 4(3) + 3}{(3)^2 - 2(3) - 3} \\ &= \frac{12 - 12}{(-9) + 9} \\ &= \frac{0}{0} \end{aligned}$$

[Since, it is of the form indeterminate]

By using factorization method:

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} &= \lim_{x \rightarrow 3} \frac{(x^2 - 4x + 3)}{(x^2 - 2x - 3)} \\ &= \lim_{x \rightarrow 3} \frac{(x^2 - 3x - x + 3)}{(x^2 - 3x + x - 3)} \\ &= \lim_{x \rightarrow 3} \frac{x(x-3) - 1(x-3)}{x(x-3) + 1(x-3)} \\ &= \lim_{x \rightarrow 3} \frac{(x-3)(x-1)}{(x-3)(x+1)} \\ &= \lim_{x \rightarrow 3} \frac{(x-1)}{(x+1)} \\ &= \frac{(3-1)}{(3+1)} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

∴ The value of the given limit is $\frac{1}{2}$.

$$3. \lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 9}$$

Solution:

Given:

The limit $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 9}$

By substituting the value of x, we get

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 9} &= \frac{(3)^4 - 81}{(3)^2 - 9} \\ &= \frac{81 - 81}{(-9) + 9} \\ &= \frac{0}{0} \text{ [Since, it is of the form indeterminate]} \end{aligned}$$

By using factorization method:

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 9} &= \lim_{x \rightarrow 3} \frac{(x^4 - 81)}{(x^2 - 9)} \\ &= \lim_{x \rightarrow 3} \frac{(x^4 - 3^4)}{(x^2 + 3^2)} \\ &= \lim_{x \rightarrow 3} \frac{((x^2)^2 - (3^2)^2)}{(x^2 - 3^2)} \text{ [Since } a^2 - b^2 = (a + b)(a - b)\text{]} \end{aligned}$$

So,

$$\begin{aligned} &= \lim_{x \rightarrow 3} \frac{(x^2 - 3^2)(x^2 + 3^2)}{(x^2 - 3^2)} \\ &= \lim_{x \rightarrow 3} (x^2 + 3^2) \\ &= 3^2 + 3^2 \\ &= 18 \end{aligned}$$

∴ The value of the given limit is 18.

$$4. \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$$

Solution:

Given:

The limit $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$

By substituting the value of x, we get

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4} &= \frac{(2)^3 - 8}{(2)^2 - 4} \\ &= \frac{8 - 8}{(4) - 4} \\ &= \frac{0}{0} \text{ [Since, it is of the form indeterminate]}\end{aligned}$$

By using factorization method:

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4} &= \lim_{x \rightarrow 2} \frac{(x^3 - 8)}{(x^2 - 4)} \\ &= \lim_{x \rightarrow 2} \frac{(x^3 - 2^3)}{(x^2 - 2^2)} \\ &= \lim_{x \rightarrow 2} \frac{(x - 2)(x^2 + 2^2 + 2x)}{(x + 2)(x - 2)}\end{aligned}$$

[By using the formula, $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$ & $(a^2 - b^2) = (a + b)(a - b)$]

$$\begin{aligned}&= \lim_{x \rightarrow 2} \frac{(x^2 + 2^2 + 2x)}{(x + 2)} \\ &= \frac{(2^2 + 2^2 + 2(2))}{(2 + 2)} \\ &= \frac{3 \cdot 4}{(4)} \\ &= 3\end{aligned}$$

∴ The value of the given limit is 3.

$$5. \lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1}$$

Solution:

Given: $\lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1}$

The limit $\lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1}$

By substituting the value of x, we get

$$\begin{aligned}\lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1} &= \frac{8\left(-\frac{1}{2}\right)^3 + 1}{2\left(-\frac{1}{2}\right) + 1} \\ &= \frac{-1 + 1}{-1 + 1} \\ &= \frac{0}{0} \text{ [Since, it is of the form indeterminate]}\end{aligned}$$

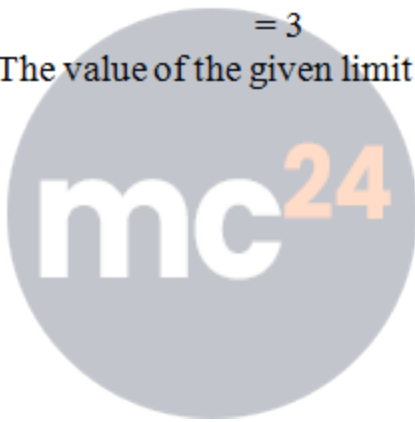
By using factorization method:

$$\begin{aligned}\lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1} &= \lim_{x \rightarrow -1/2} \frac{8x^3 + 1}{2x + 1} \\ &= \lim_{x \rightarrow -1/2} \frac{(2x)^3 + (1)^3}{2x + 1}\end{aligned}$$

[By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

$$\begin{aligned}&= \lim_{x \rightarrow -1/2} \frac{(2x + 1)((2x)^2 + (1)^2 - 2x)}{2x + 1} \\ &= \lim_{x \rightarrow -1/2} (2x)^2 + (1)^2 - 2x \\ &= \left(2\left(-\frac{1}{2}\right)\right)^2 + (1)^2 - 2\left(-\frac{1}{2}\right) \\ &= 1 + 1 + 1 \\ &= 3\end{aligned}$$

∴ The value of the given limit is 3.



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