$\frac{dy}{dx} = \frac{1}{2}$

38. Question

Mark (√) against the correct answer in the following:

If $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{-1}{(1+x^2)}$

B. $\frac{1}{(1+x^2)}$

C. $\frac{-1}{(1+x^2)^{3/2}}$

D. none of these

Answer

Given that $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$

Let x=tanθ ⇒ θ=$\tan^{-1}$x and using $\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

Hence, $y = \frac{\pi}{2} - \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4} - x\right) = \frac{1-\tan x}{1+\tan x}$, we get

$y = \frac{\pi}{2} - \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right) = \frac{\pi}{2} - \left(\frac{\pi}{4} - \theta\right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \tan^{-1}x$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{1}{1+x^2}$

39. Question

Mark (√) against the correct answer in the following:

If $y = \sqrt{\frac{1+x}{1-x}}$ then $\frac{dy}{dx} = ?$

A. $\frac{2}{(1-x)^2}$

B. $\frac{x}{(1-x)^{3/2}}$

C. $\frac{1}{(1-x)^{3/2} \cdot (1+x)^{1/2}}$

D. none of these

Answer

Given that $y=\sqrt{\frac{1+x}{1-x}}$

Let x=-cosθ ⇒ θ=$\cos^{-1}$(-x).

Using $1-\cos\theta=2\sin^2\frac{\theta}{2}$ and $1+\cos\theta=2\cos^2\frac{\theta}{2}$, we get

$$y=\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}=\tan\left(\frac{\theta}{2}\right)$$

Differentiating with respect to x, we get

$\frac{dy}{dx}=\sec^2\left(\frac{\theta}{2}\right)\times\frac{1}{2}\frac{d\theta}{dx}$ -(1)

Since, x=-cosθ ⇒ $2\cos^2\frac{\theta}{2}=1+\cos\theta=1-x$ or $\sec^2\left(\frac{\theta}{2}\right)=\frac{2}{1-x}$ -(2)

Also, since θ=$\cos^{-1}$(-x), therefore $\frac{d\theta}{dx}=\frac{1}{\sqrt{1-x^2}}$ -(3)e

Substituting (2) and (3) in (1), we get

$$\frac{dy}{dx}=\frac{2}{1-x}\times\frac{1}{2}\times\frac{1}{\sqrt{1-x^2}}=\frac{1}{(1-x)\sqrt{1-x^2}}=\frac{1}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$$

40. Question

Mark (√) against the correct answer in the following:

If $y=\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$ then $\frac{dy}{dx}=?$

A. $\frac{-2}{(1+x^2)}$

B. $\frac{2}{(1+x^2)}$

C. $\frac{-1}{(1+x^2)}$

D. none of these

Answer

Given that $y=\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

$$\Rightarrow \sec y=\frac{x^2+1}{x^2-1}$$

Since $\tan^2x=\sec^2x-1$, therefore

$$\tan^2y=\left(\frac{x^2+1}{x^2-1}\right)^2-1=\frac{4x^2}{(x^2-1)^2}$$

Hence, $\tan y=-\frac{2x}{1-x^2}$ or $y=\tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let x=tanθ ⇒ θ=$\tan^{-1}$x

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we get

$y = \tan^{-1}(-\tan 2\theta)$

Using -tan x=tan(-x), we get

$y = \tan^{-1}(\tan(-2\theta)) = -2\theta = -2\tan^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

41. Question

Mark (√) against the correct answer in the following:

If $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{-2}{(1+x^2)}$

B. $\frac{-2}{(1-x^2)}$

C. $\frac{-2}{\sqrt{1+x^2}}$

D. none of these

Answer

$\Rightarrow\ y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$\Rightarrow\ \sec y = \frac{1}{2x^2-1}$

⇒ cos y=$2x^2$ -1

⇒ y=$\cos^{-1}$ ($2x^2$ -1)

Put x = cos θ

⇒ y = $\cos^{-1}$(2 $\cos^2$ θ – 1)

⇒ y = $\cos^{-1}$(cos 2 θ)

⇒ y = 2θ

But θ = $\cos^{-1}$x.

$\Rightarrow\ \frac{dy}{dx} = \frac{d(\cos^{-1}x)}{dx}$

$\Rightarrow\ \frac{dy}{dx} = 2.\frac{d(\cos^{-1}x)}{dx}$

$$\Rightarrow \frac{dy}{dx} = 2.\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

42. Question

Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{(1+x^2)}$

B. $\frac{2}{(1+x^2)}$

C. $\frac{1}{2(1+x^2)}$

D. none of these

Answer

Put x = tan θ

$$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$\Rightarrow y = \frac{\theta}{2}$$

$\theta = \tan^{-1}x$

$$\Rightarrow y = \frac{\tan^{-1}x}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1+x^2)}$$

43. Question

Mark (√) against the correct answer in the following:

If $y = \sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$ then $\frac{dy}{dx} = ?$

A. $\frac{-1}{2\sqrt{1-x^2}}$

B. $\frac{1}{2\sqrt{1-x^2}}$

C. $\frac{1}{2\sqrt{1+x^2}}$

D. none of these

Answer

Put x = cos2θ

$\Rightarrow\ y = \sin^{-1}\left(\frac{\sqrt{1+\cos 2\theta}}{2} + \frac{\sqrt{1-\cos 2\theta}}{2}\right)$

$\Rightarrow\ y = \sin^{-1}\left(\frac{\sqrt{2\cos^2 2\theta}}{2} + \frac{\sqrt{2.\sin^2\theta}}{2}\right)$

$\Rightarrow\ y = \sin^{-1}\left(\frac{\cos 2\theta}{\sqrt{2}} + \frac{\sin 2\theta}{\sqrt{2}}\right)$

$\Rightarrow\ y = \sin^{-1}(\sin\left(\frac{\pi}{4} + 2\theta\right)$

$\Rightarrow\ y = \frac{\pi}{4} + 2\theta.$

$\Rightarrow\ \frac{dy}{d\theta} = 2$

Put $\theta = \frac{\cos^{-1}x}{2}$

$\Rightarrow\ \frac{d\theta}{dx} = \frac{-1}{4\sqrt{1-x^2}}$

$\therefore\ \frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

44. Question

Mark (√) against the correct answer in the following:

If $x = at^2, y = 2at$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{t}$

B. $\frac{-1}{t^2}$

C. $\frac{-2}{t}$

D. none of these

Answer

x = at^2

$\therefore \frac{dx}{dt} = 2at$

$\therefore \frac{dt}{dx} = \frac{1}{2at}$

Y = 2at

$\therefore \frac{dy}{dt} = 2a$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = 2a \times \frac{1}{2at}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{t}$

45. Question

Mark (√) against the correct answer in the following:

If $x = a\sec\theta, y = b\tan\theta$ then $\frac{dy}{dx} = ?$

A. $\frac{b}{a}\sec\theta$

B. $\frac{b}{a}\operatorname{cosec}\theta$

C. $\frac{b}{a}\cot\theta$

D. none of these

Answer

x = a sec θ

$\therefore \frac{dx}{d\theta} = a\sec\theta.\tan\theta$

$\therefore \frac{d\theta}{dx} = \frac{1}{a\sec\theta.\tan\theta}$

y = b tan θ

$\therefore \frac{dy}{d\theta} = b.\sec^2\theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx} = b.\sec^2\theta \times \frac{1}{a\sec\theta.\tan\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{b\sec\theta}{a\tan\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{b.\frac{1}{\cos\theta}}{a.\frac{\sin\theta}{\cos\theta}}$

$\Rightarrow \frac{dy}{dx} = \frac{b}{a}\csc\theta$

46. Question

Mark (√) against the correct answer in the following:

If $x = a\cos^2\theta, y = b\sin^2\theta$ then $\frac{dy}{dx} = ?$

A. $\frac{-a}{b}$

B. $\frac{-a}{b}\cot\theta$

C. $\frac{-b}{a}$

D. none of these

Answer

x = a.cos$^2\theta$

$\therefore \frac{dx}{d\theta} = -2a\cos\theta.\sin\theta$

$\Rightarrow \frac{d\theta}{dx} = \frac{-1}{2a.\cos\theta.\sin\theta}$

y = b.sin$^2\theta$

$\therefore \frac{dy}{d\theta} = 2b\sin\theta.\cos\theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx} = 2b\sin\theta.\cos\theta \times \frac{-1}{2a\cos\theta.\sin\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{-b}{a}$

47. Question

Mark (√) against the correct answer in the following:

If $x = \theta(\cos\theta + \sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ then $\frac{dy}{dx} = ?$

A. cot θ

B. tan θ

C. a cot θ

D. a tan θ

Answer

x = a(cos θ + θ sin θ)

$\therefore \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$\Rightarrow \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}$

y = a(sin θ – θ cos θ)

$\therefore \frac{dy}{d\theta} = a(\cos\theta - (\cos\theta + \theta(-\sin\theta)))$

$\Rightarrow \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + \theta a\sin\theta$

$\Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx} = a\theta\sin\theta \times \frac{1}{a\theta\cos\theta}$

$\Rightarrow \frac{dy}{dx} = \tan\theta$

48. Question

Mark (√) against the correct answer in the following:

If $y = x^{x^{x\ldots\infty}}$ then $\frac{dy}{dx} = ?$

A. $\frac{y}{x(1-\log x)}$

B. $\frac{y^2}{x(1-\log x)}$

C. $\frac{y}{x(1-y\log x)}$

D. none of these

Answer

Given:

$\Rightarrow y = x^{x^{x^{x\ldots\infty}}}$

We can write it as

$\Rightarrow y = x^y$

Taking log of both sides we get

log y = y log x

Differentiating

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x + y.\frac{1}{x}$

$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y}{1 - \log x}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1 - \log x)}$

49. Question

Mark (√) against the correct answer in the following:

If $y = \sqrt{x + \sqrt{x + \sqrt{x +}}} \ldots\infty$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{(2y - 1)}$

B. $\frac{1}{(y^2 - 1)}$

C. $\frac{2y}{(y^2 - 1)}$

D. none of these

Answer

Given:

$\Rightarrow y = \sqrt{x + \sqrt{x + \sqrt{x +}}} \ldots\infty$

We can write it as

$\Rightarrow y = \sqrt{x + y}$

Squaring we get

$\Rightarrow y^2 = x + y$

Differentiating

$\Rightarrow 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{(2y - 1)}$

50. Question

Mark (√) against the correct answer in the following:

If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x +}}} \ldots\infty$ then $\frac{dy}{dx} = ?$

A. $\frac{\sin x}{(2y - 1)}$

B. $\frac{\cos x}{(y-1)}$

C. $\frac{\cos x}{(2y-1)}$

D. none of these

Answer

Given:

$\Rightarrow y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\infty}}}$

We can write it as

$\Rightarrow y = \sqrt{\sin x + y}$

Squaring we get

$\Rightarrow y^2 = \sin x + y$

Differentiating

$\Rightarrow 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{(2y-1)}$

51. Question

Mark (√) against the correct answer in the following:

If $y = e^x + e^{x+\ldots\infty}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{(1-y)}$

B. $\frac{y}{(1-y)}$

C. $\frac{y}{(y-1)}$

D. none of these

Answer

We can write it as

$\Rightarrow y = e^{x+y}$

$\log y = (x + y) \log e$

Differentiating

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \left(\frac{1}{y}-1\right)\frac{dy}{dx}=1$

$\Rightarrow \frac{dy}{dx}=1\left(\frac{y}{1-y}\right)$

52. Question

Mark (√) against the correct answer in the following:

The value of k for which $f(x)=\begin{cases}\frac{\sin 5x}{3x}, \text{if } x\neq 0\\ k, \text{if } x=0\end{cases}$ is continuous at x = 0 is

A. $\frac{1}{3}$

B. 0

C. $\frac{3}{5}$

D. $\frac{5}{3}$

Answer

Since f(x) is continuous on 0.

$\Rightarrow \lim_{x\to 0}\frac{\sin 5x}{3x}=f(0)$

$\Rightarrow \lim_{x\to 0}\frac{\sin 5x}{3x}\times\frac{5x}{5x}=f(0)$

$\Rightarrow \lim_{x\to 0}\frac{\sin 5x}{5x}\times\frac{5x}{3x}=f(0)$

$\Rightarrow f(0)=\frac{5}{3}$

$\Rightarrow k=\frac{5}{3}$

53. Question

Mark (√) against the correct answer in the following:

Let $f(x)=\begin{cases}x\sin\frac{1}{x}, \text{ if } x\neq 0\\ 0, \text{ when } x=0.\end{cases}$

Then, which of the following is the true statement?

A. $f(x)$ is not defined at x = 0

B. $\lim_{x\to 0} f(x)$ does not exist

C. $f(x)$ is continuous at x = 0

D. $f(x)$ is discontinuous at x = 0

Answer

Left hand limit =

$\Rightarrow \lim\limits_{x\to 0^-} f(x)$

$\Rightarrow \lim\limits_{h\to 0} f(0-h)$

$\Rightarrow \lim\limits_{h\to 0} h.\sin\left(\frac{-1}{h}\right)$

$\Rightarrow \lim\limits_{h\to 0} -h.\frac{\sin\left(\frac{-1}{h}\right)}{-\frac{1}{h}} \times \frac{-1}{h} = 1$

Right hand limit =

$\Rightarrow \lim\limits_{x\to 0^+} f(x)$

$\Rightarrow \lim\limits_{h\to 0} f(0+h)$

$\Rightarrow \lim\limits_{h\to 0} h.\sin\left(\frac{1}{h}\right)$

$\Rightarrow \lim\limits_{h\to 0} h.\frac{\sin\left(\frac{1}{h}\right)}{\frac{1}{h}} \times \frac{1}{h}$

= 1

As L.H.L = R.H.L

F(x) is continuous.

54. Question

Mark (√) against the correct answer in the following:

The value of k for which $f(x) = \begin{cases} \frac{3x + 4\tan x}{2}, \text{ when } x \neq 0 \\ k, \text{ when } x = 0 \end{cases}$ is continuous at x = 0, is

A. 7

B. 4

C. 3

D. none of these

Answer

$\Rightarrow f(x) = \frac{3x+4\tan x}{x}$ is continuous at x = 0.

$\Rightarrow f(x) = \lim\limits_{x\to 0} \frac{3x + 4\tan x}{x}$

$\Rightarrow f(x) = \lim\limits_{x\to 0} \frac{3x}{x} + \frac{4\tan x}{x}$

$\Rightarrow f(x) = 3 + 4\lim\limits_{x\to 0} \frac{\tan x}{x}$

$\Rightarrow$ f(x) = 3+4

∴ K = 7.

55. Question

Mark (√) against the correct answer in the following:

Let $f(x)=x^{3/2}$. Then, $f'(0)=?$

A. $\frac{3}{2}$

B. $\frac{1}{2}$

C. does not exist

D. none of these

Answer

$f(x) = x^{3/2}$

$\Rightarrow f'(x)=\frac{3}{2\sqrt{x}}$

As $x \to 0$, $f'(x) \to \infty$

∴ f'(x) does not exist.

56. Question

Mark (√) against the correct answer in the following:

The function $f(x)=|x| \ \forall x \in R$ is

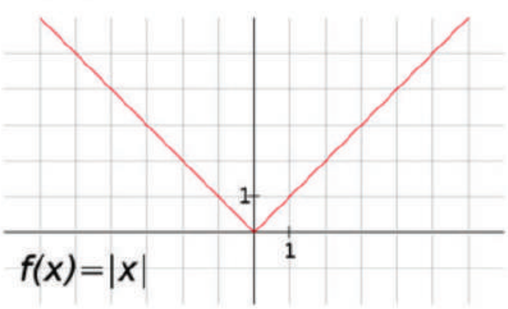
A. continuous but not differentiable at x = 0

B. differentiable but not continuous at x = 0

C. neither continuous nor differentiable at x = 0

D. none of these

Answer



(Sometimes it's easier to get the answer by graphs)

Now in the above graph

We can see f(x) is Continuous on 0.

But it has sharp curve on x = 0 which implies it is not differentiable.

57. Question

Mark (√) against the correct answer in the following:

The function $f(x)=\begin{cases}1+x, \text{ when } x \le 2\\ 5-x, \text{ when } x>2\end{cases}$ is

A. continuous as well as differentiable at x = 2

B. continuous but not differentiable at x = 2

C. differentiable but not continuous at x = 2

D. none of these

Answer

For continuity left hand limit must be equal to right hand limit and value at the point.

Continuity at x =2.

For continuity at x=2,

L.H.L = $\lim_{x\to 2^-}(1+x) = 3$

R.H.L = $\lim_{x\to 2^+}(5-x) = 3$

f(2) = 1+2 = 3

∴ f(x) is continuous at x = 2

Now for differentiability.

$\Rightarrow f'(2^-) = \lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2}$

$\Rightarrow f'(2^-) = \lim_{h\to 0}\frac{f(2-h)-f(2)}{2-h-2}$

$\Rightarrow f'(2^-) = \lim_{h\to 0}\frac{1+2-h-3}{2-h-2} = \lim_{h\to 0}\frac{-h}{-h} = 1.$

$\Rightarrow f'(2^+) = \lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2}$

$\Rightarrow f'(2^+) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{2+h-2}$

$\Rightarrow f'(2^-) = \lim_{h\to 0}\frac{5-(2-h)-3}{2+h-2}$

$= \lim_{h\to 0}\frac{h}{-h}$

=-1

As, $f'(2^-)$ is not equal to $f(2^+)$

∴ f(x) is not differentiable.

58. Question

Mark (√) against the correct answer in the following:

If $f(x)=\begin{cases}kx+5, \text{ when } x \le 2\\ x+1, \text{ when } x>2\end{cases}$ is continuous at x = 2 then k = ?

A. 2

B. -2

C. 3

D. -3

Answer

For continuity left hand limit must be equal to right hand limit and value at the point.

Continuous at x =2.

L.H.L = $\lim_{x \to 2^-}(kx + 5)$

$\Rightarrow \lim_{h \to 0}(k(2 - h) + 5)$

⇒ k(2-0)+5 = 2k+5

R.H.L = $\lim_{x \to 2^+}(x + 1)$

$\Rightarrow \lim_{h \to 0}(2 + h + 1)$

⇒ 2+0+1

=3

As f(x) is continuous

∴ 2k+5 = 3

K = -1.

59. Question

Mark (√) against the correct answer in the following:

If the function $f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, x \neq 0 \\ k, x = 0 \end{cases}$ is continuous at x = 0 and then k = ?

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

Answer

Given:

$\Rightarrow f(x) = \frac{1 - \cos 4x}{8x^2}$ is continuous at x = 0.

⇒ 1-cos4x = $2\sin^2 2x$

$\Rightarrow f(x) = \lim_{x \to 0}\frac{2\sin^2 2x}{8x^2}$

$\Rightarrow f(x) = \lim_{x \to 0}\frac{2\sin^2 2x}{2 \times 4x^2}$

$\Rightarrow f(x) = \lim_{x \to 0}\left(\frac{\sin 2x}{2x}\right)^2$

$\Rightarrow f(x) = 1$

$\therefore$ Kk= 1

60. Question

Mark (√) against the correct answer in the following:

If the function $f(x)=\begin{cases}\dfrac{\sin^2 ax}{x^2}, \text{ when } x\neq 0\\ k, \text{ when } x=0\end{cases}$ is continuous at x = 0 then k = ?

A. a

B. a^2

C. -2

D. -4

Answer

F(x) is continuous at x = 0.

$\Rightarrow f(x) = \lim\limits_{x\to 0}\dfrac{\sin^2 ax}{x^2}$

$\Rightarrow f(x) = \lim\limits_{x\to 0}\dfrac{\sin^2 ax}{x^2}\times\dfrac{a^2}{a^2}$

$\Rightarrow f(x) = \lim\limits_{x\to 0}\left(\dfrac{\sin ax}{ax}\right)^2\times a^2$

$\Rightarrow f(x) = a^2$

$\therefore k = a^2$

61. Question

Mark (√) against the correct answer in the following:

If the function $f(x)=\begin{cases}\dfrac{k\cos x}{(\pi-2x)}, \text{ when } x\neq\dfrac{\pi}{2}\\ 3, \text{ when } x=\dfrac{\pi}{2}\end{cases}$ be continuous at $x=\dfrac{\pi}{2}$, then the value of k is

A. 3

B. -3

C. -5

D. 6

Answer

Given: f(x) is continuous at x= π/2.

$\therefore$ L.H.L $= \lim\limits_{x\to\frac{\pi}{2}^-} f(x)$

$= \lim\limits_{x\to\frac{\pi}{2}^-}\dfrac{k\cos x}{\pi-2x}$

Putting $x = \frac{\pi}{2} - h$;

As $x \to \frac{\pi}{2}^-$ then $h \to 0$.

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = k.\lim_{h \to 0} \frac{\sin h}{h}$$

∴ L.H.L = k

As it is continuous which implies right hand limit equals left hand limit equals the value at that point.

∴ k=3.

62. Question

Mark (√) against the correct answer in the following:

At $x = 2,\ f(x) = |x|$ is

A. continuous but not differentiable

B. differentiable but not continuous

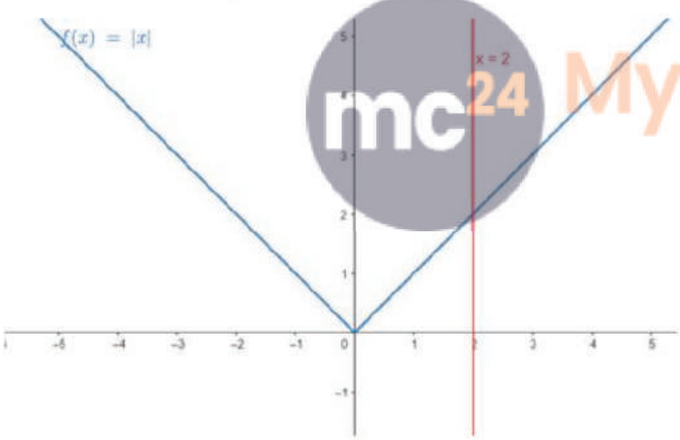
C. continuous as well as differentiable

D. none of these

Answer

Given:

Let us see that graph of the modulus function.



We can see that f(x) = |x| is neither continuous and nor differentiable at x = 2. Hence, D is the correct answer.

63. Question

Mark (√) against the correct answer in the following:

Let $f(x) = \begin{cases} \frac{x^2 - 2x - 3}{x + 1^2}, & \text{when } x \neq -1 \\ k, & \text{when } x = -1 \end{cases}$

If $f(x)$ is continuous at x = -1 then k = ?

A. 4

B. -4

C. -3

D. 2

Answer

$\Rightarrow f(x) = \frac{x^2-2x-3}{x+1}$ is continuous at x = 0.

$\Rightarrow f(x) = \lim_{x \to -1} \frac{(x+1)(x-3)}{x+1}$

$\Rightarrow f(x) = \lim_{x \to -1} x - 3$

$\Rightarrow f(x) = -4$

$\therefore K = 1.$

64. Question

Mark (√) against the correct answer in the following:

The function $f(x) = x^3 + 6x^2 + 15x - 12$ is

A. strictly decreasing on R

B. strictly increasing on R

C. increasing in $(-\infty, 2)$ and decreasing in $(2, \infty)$

D. none of these

Answer

Given:

$f(x) = x^3+6x^2+15x-12.$

$f'(x) = 3x^2+12x+15$

$f'(x) = 3x^2+12x+12+3$

$f'(x) = 3(x^2+4x+4)+3$

$f'(x) = 3(x+2)^2+3$

As square is a positive number

$\therefore$ f'(x) will be always positive for every real number

Hence f'(x) >0 for all x ∈ R

$\therefore$ f(x) is strictly increasing.

65. Question

Mark (√) against the correct answer in the following:

The function $f(x) = 4 - 3x + 3x^2 - x^3$ is

A. decreasing on R

B. increasing on R

C. strictly decreasing on R

D. strictly increasing on R

Answer

$f(x) = -x^3+3x^2-3x+4.$

$f'(x) = -3x^2+6x-3$

$f'(x) = -3(x^2-2x+1)$

$f'(x) = -3(x-1)^2$

As f'(x) has –ve sign before 3

⇒ f'(x) is decreasing over R.

66. Question

Mark (√) against the correct answer in the following:

The function $f(x) = 3x + \cos 3x$ is

A. increasing on R

B. decreasing on R

C. strictly increasing on R

D. strictly decreasing on R

Answer

Given:

$f(x) = 3x+\cos 3x$

$f'(x) = 3-3\sin 3x$

$f'(x) = 3(1-\sin 3x)$

sin3x varies from[-1,1]

when sin3x is 1 f'(x) = 0 and sin3x is -1 f'(x) = 6

As the function is increasing in 0 to 6.

∴ The function is increasing on R.

67. Question

Mark (√) against the correct answer in the following:

The function $f(x) = x^3 + 6x^2 + 9x + 3$ is decreasing for

A. $1 < x < 3$

B. $x > 1$

C. $x < 1$

D. $x < 1$ or $x > 3$

Answer

Given:

$f(x) = x^3+6x^2+9x+3.$

$f'(x) = 3x^2+12x+9 = 0$

$f'(x) = 3(x^2+4x+3) =0$

$f'(x) = 3(x+1)(x+3) = 0$

$x = -1$ or $x =-3$

for $x>-1$ f(x) is increasing

for $x<-3$ f(x) is increasing

But for -1<x<-3 it is decreasing.

68. Question

Mark (√) against the correct answer in the following:

The function $f(x) = x^3 - 27x + 8$ is increasing when

A. $|x| < 3$

B. $|x| > 3$

C. -3 < x < 3

D. none of these

Answer

Given:

$f(x) = x^3 - 27x + 8$.

$f'(x) = 3x^2 - 27x = 0$

$f'(x) = 3(x^2 - 9) = 0$

$f'(x) = 3(x-3)(x+3) = 0$

x = 3 or x =-3

for x>3 f(x) is increasing

for x<-3 f(x) is increasing

∴ for |x|>3 f(x) is increasing.

69. Question

Mark (√) against the correct answer in the following:

$f(x) = \sin x$ is increasing in

A. $\left(\frac{\pi}{2}, \pi\right)$

B. $\left(\pi, \frac{3\pi}{2}\right)$

C. $(0, \pi)$

D. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Answer

Given: f(x) is sin x

∴ f'(x) = cos x

⇒ f'(x) =cos x

= 0

⇒ $\text{for } x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

f'(x) is increasing

$\therefore$ f(x) is increasing in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

70. Question

Mark (√) against the correct answer in the following:

$f(x) = \frac{2x}{\log x}$ is increasing in

A. (0, 1)

B. (1, e)

C. (e, ∞)

D. (-∞, e)

Answer

$\Rightarrow f(x) = \frac{2x}{\log x}$

$\Rightarrow f'(x) = \frac{2.\log x - 2}{\log^2 x}$

Put f'(x) = 0

We get

$\Rightarrow \frac{2.\log x - 2}{\log^2 x} = 0$

⇒ 2.logx = 2

log x = 1

⇒ x= e

We only have one critical point

So, we can directly say x>e f(x) would be increasing

∴ f(x) will be increasing in (e, ∞)

71. Question

Mark (√) against the correct answer in the following:

$f(x) = (\sin x - \cos x)$ is decreasing in

A. $\left(0, \frac{3\pi}{4}\right)$

B. $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

C. $\left(\frac{7\pi}{4}, 2\pi\right)$

D. none of these

Answer

Given:

f(x) = sin x – cos x

f′(x) = cos x + sin x

Multiply and divide by √2.

$$\Rightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$$

$$\Rightarrow \sqrt{2}\left(\sin\frac{\pi}{4}.\cos x + \cos\frac{\pi}{4}.\sin x\right)$$

$$\Rightarrow \sqrt{2}(\sin\left(\frac{\pi}{4} + x\right))$$

$$\Rightarrow f'(x) = \sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$$

For f(x) to be decreasing .f′(x)<0

$$\Rightarrow f'(x) = \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) < 0$$

$$\Rightarrow \pi < x + \frac{\pi}{4} < 2\pi$$

(∵ sin θ <0 for π <θ <2π)

$$\Rightarrow \pi - \frac{\pi}{4} < x < 2\pi - \frac{\pi}{4}$$

$$\Rightarrow \frac{3\pi}{4} < x < \frac{7\pi}{4}$$

∴ f(x) decreases in the interval.

$$\Rightarrow \left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$$

72. Question

Mark (√) against the correct answer in the following:

$f(x) = \frac{x}{\sin x}$ is

A. increasing in (0, 1)

B. decreasing in (0, 1)

C. increasing in $\left(0, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, 1\right)$

D. none of these

Answer

$$\Rightarrow f(x) = \frac{x}{\sin x}$$

$$\Rightarrow f'(x) = \frac{\sin x - x\cos x}{\sin x}$$

Now see

In (0,1) sin x is increasing and cos x is decreasing

sin x – x cos x will be increasing

∴ f(x) is increasing in (0,1)

73. Question

Mark (√) against the correct answer in the following:

$f(x) = x^x$ is decreasing in the interval

A. (0, e)

B. $\left(0, \frac{1}{e}\right)$

C. (0,1)

D. none of these

Answer

Given: f(x) = x^x.

⇒ f'(x)=(log x+1) x^x

⇒ keeping f'(x) = 0

We get

⇒ $x = 0$ or $x = \frac{1}{e}$

Now

When x>1/e the function is increasing

x<0 function is increasing.

But in the interval (0,1/e) the function is decreasing.

74. Question

Mark (√) against the correct answer in the following:

$f(x) = x^2e^{-x}$ is increasing in

A. (-2, 0)

B. (0, 2)

C. (2, ∞)

D. (-∞, ∞)

Answer

Given f(x) = $x^2.e^{-x}$

⇒ f'(x) = 2x. e^{-x} – x^2 e^{-x}

⇒ Put f'(x) = 0

⇒ - (x^2– 2x)e^{-x} = 0

⇒ x = 0 or x =2.

Now as there is a -ve sign before f'(x)

When x>2 the function is decreasing

x<0 function is decreasing

But in the interval (0,2) the function is increasing.

75. Question

Mark (√) against the correct answer in the following:

$f(x) = \sin x - kx$ is decreasing for all $x \notin R$, when

A. $k < 1$

B. $k \le 1$

C. $k > 1$

D. $k \le 1$

Answer

$f(x) = \sin x - kx$

$f'(x) = \cos x - k$

$\therefore$ f decreases, if $f'(x) \le 0$

$\Rightarrow \cos x - k \le 0$

$\Rightarrow \cos x \le k$

So, for decreasing $k \ge 1$.

76. Question

Mark (√) against the correct answer in the following:

$f(x) = (x+1)^3(x-3)^3$ is increasing in

A. $(-\infty, 1)$

B. $(-1, 3)$

C. $(3, \infty)$

D. $(1, \infty)$

Answer

Given:

$\Rightarrow f(x) = (x+1)^3.(x-3)^3$

$\Rightarrow f'(x) = 3(x+1)^2(x-3)^3 + 3(x-3)^3(x+1)^3$

Put $f'(x) = 0$

$\Rightarrow 3(x+1)^2(x-3)^3 = -3(x-3)^2(x+1)^3$

$\Rightarrow x-3 = -(x+1)$

$\Rightarrow 2x = 2$

$\Rightarrow x = 1$

When $x>1$ the function is increasing.

$x<1$ function is decreasing.

So, f(x) is increasing in $(1, \infty)$.

77. Question

Mark (√) against the correct answer in the following:

$f(x) = [x(x-3)]^2$ is increasing in

A. $(0, \infty)$

B. $(-\infty, 0)$

C. (1, 3)

D. $\left(0, \frac{3}{2}\right) \cup (3, \infty)$

Answer

$\Rightarrow f(x)=[x(x-3)]^2$

$\Rightarrow f'(x)=2[x(x-3)]=0$

$\Rightarrow x = 3 \text{ and } x = \frac{3}{2}$

When x> 3/2 the function is increasing

X<3 function is increasing.

$\Rightarrow \left(0, \frac{3}{2}\right) \cup (3, \infty)$ Function is increasing.

78. Question

Mark (√) against the correct answer in the following:

If $f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing for every real number x, then

A. $k > 3$

B. $k \geq 3$

C. $k < 3$

D. $k \leq 3$

Answer

Given $f(x) = kx^3 - 9x^2 + 9x + 3$

$\Rightarrow f'(x) = 3kx^2 - 18x + 9$

$\Rightarrow f'(x) = 3(kx^2 - 6x + 3) > 0$

$\Rightarrow kx^2 - 6x + 3 > 0$

For quadratic equation to be greater than 0. a>0 and D<0.

$\Rightarrow k>0$ and $(-6)^2 - 4(k)(3)<0$

$\Rightarrow 36 - 12k<0$

$\Rightarrow 12k>36$

$\Rightarrow k>3$

$\therefore k>3$.

79. Question

Mark (√) against the correct answer in the following:

$f(x) = \frac{x}{(x^2 - 1)}$ is increasing in

A. (-1, 1)

B. (-1, ∞)

C. $(-\infty, -1) \cup (1, \infty)$

D. none of these

Answer

$\Rightarrow f(x) = \frac{x}{x^2 + 1}$

$\Rightarrow f'(x) = \frac{x^2 - 2x^2 + 1}{x^2 + 1}$

$\Rightarrow f'(x) = -\frac{x^2 - 1}{x^2 + 1}$

⇒ For critical points f'(x) = 0

When f'(x) = 0

We get x = 1 or x = -1

When we plot them on number line as f'(x) is multiplied by –ve sign we get

For x>1 function is decreasing

For x<-1 function is decreasing

But between -1 to 1 function is increasing.

∴ Function is increasing in(-1,1).

80. Question

Mark (√) against the correct answer in the following:

The least value of k for which $f(x) = x^2 + kx + 1$ is increasing on (1, 2), is

A. -2

B. -1

C. 1

D. 2

Answer

f(x) $=x^2+kx+1$

For increasing

f'(x) = 2x+k

k≥ -2x

thus,

k≥ -2.

Least value of -2.

81. Question

Mark (√) against the correct answer in the following:

$f(x) = |x|$ has

A. minimum at x = 0

B. maximum x = 0

C. neither a maximum nor a minimum at x = 0

D. none f these

Answer

f(x) = |x|

Now to check the maxima and minima at x =0.

It can be easily seen through the option.

See |x| is x for x>0 and –x for x<0

That is no matter if you put a number greater than zero or number less than zero you will get positive answer.

∴ for x = 0 we will get minima.

82. Question

Mark (√) against the correct answer in the following:

When x is positive, the minimum value of x^x is

A. e^e

B. $e^{1/e}$

C. $e^{-1/e}$

D. $(1/e)$

Answer

Given: $f(x) = x^x$.

⇒ $f'(x) = (\log x + 1)\, x^x$

⇒ keeping $f'(x) = 0$

We get

$$\Rightarrow x = 0 \text{ or } x = \frac{1}{e}$$

$$\Rightarrow f''(x) = x^x(1 + \log x)\left[1 + \log x + \frac{1}{x(1 + \log x)}\right]$$

When x is greater than zero,

We get a maximum value as the function will be negative.

Therefore,

$F(x) = x^x$

$F(e) = \left(\frac{1}{e}\right)^{1/e} = e^{-\frac{1}{e}}$

Hence, C is the correct answer.

83. Question

Mark (√) against the correct answer in the following:

The maximum value of $\left(\frac{\log x}{x}\right)$ is

A. $\left(\frac{1}{e}\right)$

B. $\frac{2}{e}$

C. e

D. 1

Answer

$\Rightarrow f(x) = \frac{\log x}{x}$

$\therefore f'(x) = \frac{\log x - x\frac{1}{x}}{x^2}$

⇒ f'(x) = log x – 1

⇒ Put f'(x) = 0

We get x =e

F''(x) = 1/x

Put x =e in f''(X)

1/e is point of maxima

∴ The max value is 1/e.

84. Question

Mark (√) against the correct answer in the following:

$f(x) = \operatorname{cosec} x$ in (-π, 0) has a maxima at

A. x = 0

B. $x = \frac{-\pi}{4}$

C. $x = \frac{-\pi}{3}$

D. $x = \frac{-\pi}{2}$

Answer

We can go through options for this question

Option a is wrong because 0 is not included in (-π,0)

At x = -π/4 value of f(x) is -√2 = -1.41

At x = -π/3 value of f(x) is -2.

At x = -π/2 value of f(x) is -1.

∴ f(x) has max value at x =-π/2.

Which is -1.

85. Question

Mark (√) against the correct answer in the following:

If x > 0 and xy = 1, the minimum value of (x + y) is

A. -2

B. 1

C. 2

D. none of these

Answer

Given: x>0 and xy = 1

We need to find the minimum value of (x + y).

$\Rightarrow y = \frac{1}{x}$

$\Rightarrow f(x) = x + \frac{1}{x}$

$\Rightarrow f(x) = \frac{x^2 + 1}{x}$

$\Rightarrow f'(x) = \frac{x.2x - (x^2 + 1).1}{x^2}$

$\Rightarrow f'(x) = \frac{x^2 - 1}{x^2}$

$\Rightarrow f''(x) = \frac{x^2(2x) - (x^2 - 1).2x}{x^4}$

$\Rightarrow f''(x) = \frac{2x}{x^4}$

$\Rightarrow f''(x) = \frac{2}{x^3}$

For maximum or minimum value f'(x) = 0.

$\therefore \frac{x^2 - 1}{x^2} = 0$

∴ x = 1 or x =-1

f''(x) at x = 1.

∴ f''(x) = 2.

F''(x)>0 it is decreasing and has minimum value at x = 1

At x = -1

f''(x) = -2

f''(x)<0 it is increasing and has maximum value at x = -1.

∴ Substituting x = 1 in f(x) we get

f(x) = 2.

∴ The minimum value of given function is 2.

86. Question

Mark (√) against the correct answer in the following:

The minimum value of $\left(x^2+\frac{250}{x}\right)$ is

A. 0

B. 25

C. 50

D. 75

Answer

$\Rightarrow f(x) = x^2+\frac{250}{x}$

$\Rightarrow f'(x) = 2x-\frac{250}{x^2} = 0$

$\Rightarrow 2x^3 = 250$

$\Rightarrow x^3 = 125$

$\Rightarrow x = 5$

Substituting x = 5 in f(x) we get

f(x) = 25+50

f(x) = 75.

87. Question

Mark (√) against the correct answer in the following:

The minimum value of $f(x) = 3x^4-8x^3-48x+25$ on [0, 3] is

A. 16

B. 25

C. -39

D. none of these

Answer

Given:

$f(x) = 3x^4-8x^3-48x+25$.

$F'(x) = 12x^3-24x^2-48 = 0$

$F'(x) = 12(x^3-2x^2-4) = 0$

Differentiating again, we get,

$F''(x) = 3x^2 - 4x = 0$

$x(3x - 4) = 0$

x = 0 or x = 4/3

Putting the value in equation, we get,

$f(x) = -39$

Hence, C is the correct answer.

88. Question

Mark (√) against the correct answer in the following:

The maximum value of $f(x)=(x-2)(x-3)^2$ is

A. $\frac{7}{3}$

B. 3

C. $\frac{4}{27}$

D. 0

Answer

$f(x) = (x-2)(x-3)^2$

$f(x) = (x-2)(x^2-6x+9)$

$f(x) = x^3-8x^2+21x-18.$

$f'(x) = 3x^2-16x+21$

$f''(x) = 6x-16$

For maximum or minimum value $f'(x) = 0$.

$\therefore 3x^2-9x-7x+21 = 0$

$\Rightarrow 3x(x-3)-7(x-3)=0$

$\Rightarrow x = 3$ or $x = 7/3$.

$f''(x)$ at $x = 3$.

$\therefore f''(x) = 2$

$f''(x)>0$ it is decreasing and has minimum value at $x = 3$

At $x = 7/3$

$F''(x) = -2$

$F''(x)<0$ it is increasing and has maximum value at $x = 7/3$.

Substituting $x = 7/3$ in $f(x)$ we get

$\Rightarrow \left(\frac{7}{3}-2\right)\left(\frac{7}{3}-3\right)^2$

$\Rightarrow \left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)^2$

$\Rightarrow \frac{4}{27}$

89. Question

Mark (√) against the correct answer in the following:

The least value of $f(x) = \left(e^{x} + e^{-x}\right)$ is

A. -2

B. 0

C. 2

D. none of these

Answer

$f(x) = e^{x} + e^{-x}$

$\Rightarrow f(x) = e^{x} + \frac{1}{e^{x}}$

$\Rightarrow f(x) = \frac{e^{2x} + 1}{e^{x}}$

f(x) is always increasing at x = 0it has the least value

$\Rightarrow f(x) = \frac{1 + 1}{1} = 2$

$\therefore$ The least value is 2.