NCERT Solutions for Class-XII Maths

Chapter-7.1

NCERT Maths Class 12

1. Find an anti-derivative (or integral) of the following functions by the method of inspection. sin 2x

1. The anti-derivative of sin 2x is a function of x whose derivative is sisn 2x. It is known that,

$$\frac{d}{dx}(\cos 2x) = 2\sin 2x$$

$$\Rightarrow \sin 2x - \frac{1}{12}\frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)s$$

Therefore, the anti-derivative of sin 2x is $-\frac{1}{2}\cos 2x$

2. Find an anti-derivative (or integral) of the following functions by the method of inspection. cos 3x.

2. We know that $\frac{d(\sin 3x)}{dx} = 3\cos 3x$

$$\Rightarrow 3x = \frac{1}{3}\frac{d(\sin 3x)}{dx}$$

$$= \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Therefore, the anti-derivative of $\cos^3 x$ is $\frac{1}{3}\sin 3x$.

3. Find an anti-derivative (or integral) of the following functions by the method of inspection. e^{2x}

3. The anti-derivative of e^{2x} is the function of x whose derivative is e^{2x}.
It is known that,

$$\frac{d}{dx}\left(e^{2x}\right) = 2e^{2x}$$

$$\Rightarrow e^{2x} = \frac{1}{d}\frac{d}{dx}\left(e^{2x}\right)$$

$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$

Therefore, the anti-derivative of e^{2x} is $\frac{1}{2}e^{2x}$

4. Find an anti-derivative (or integral) of the following functions by the method of inspection. $(ax + b)^2$

4. We know that $\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$

$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d(ax+b)^3}{dx}$

$= \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$

Therefore, the anti-derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

5. Find an anti-derivative (or integral) of the following functions by the method of inspection. $\sin 2x - 4e^{3x}$

5. The anti-derivative of $\sin 2x - 4e^{3x}$ is the function of x whose derivative is $\sin 2x - 4e^{3x}$

It is known that,

$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$

Therefore, the anti-derivative of $\left(\sin 2x - 4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$

6. $\int\left(4e^{3x}+1\right)dx$

6. $\int\left(4e^{3x}+1\right)dx = 4\int e^{3x}dx + \int 1dx$

$= 4\left(\frac{e^{3x}}{3}\right) + x + C$

$= \frac{4}{3}e^{3x} + x + C$

7. $\int x^2\left(1-\frac{1}{x^2}\right)dx$

7. $\int x^2\left(1-\frac{1}{x^2}\right)dx$

$= \int\left(x^2-1\right)dx$

$= \frac{x^3}{3} - x + C$

8. $\int (ax^2 + bx + c)dx$

8. $\int (ax^2 + bx + c)dx$

$= a\int x^2 dx + b\int x dx + c\int 1.dx$

$= a\left(\frac{x^3}{3}\right) + b\left(\frac{x^2}{2}\right) + cx + C$

$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$

9. $\int (2x^2 + e^x)dx$

9. $\int (2x^2 + e^x)dx$

$= 2\int x^2 dx + \int e^x dx$

$= 2\left(\frac{x^3}{3}\right) + e^x + C$

$= \frac{2}{3}x^3 + e^x + C$

10. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

10. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

$= \int \left(x + \frac{1}{x} - 2\right)dx$

$= \int x dx + \int \frac{1}{x} dx - 2\int 1.dx$

$= \frac{x^2}{2} + \log|x| - 2x + C.$

11. $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$

11. $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$

$= \int (x + 5 - 4x^{-2})dx$

$$= \int x dx + 5\int 1.dx - 4\int x^{-2} dx$$

$$= \frac{x^2}{2} + 5x - 4\left(\frac{x^{-1}}{-1}\right) + C$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

12. $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

12. $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

$$= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + \frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}} + C$$

$$= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$$

$$= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$$

13. $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

13. $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

On dividing, we obtain

$$= \int \left(x^2 + 1\right) dx$$

$$= \int x^2 dx + \int 1 dx$$

$$= \frac{x^3}{3} + x + C$$

14. $\int (1 - x)\sqrt{x} dx$

14. $\int (1 - x)\sqrt{x} dx$

$$= \int \left(\sqrt{x} - x^{\frac{3}{2}}\right) dx$$

$$= \int x^{\frac{1}{2}} dx - \int x^{\frac{3}{2}} dx$$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$

$= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$

15. $\int \sqrt{x}\left(3x^2 + 2x + 3\right)dx$

15. $\int \sqrt{x}(3x^2 + 2x + 3)dx$

$= \int \left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right)dx$

$= 3\int x^{\frac{5}{2}}dx + 2\int x^{\frac{3}{2}}dx + 3\int x^{\frac{1}{2}}dx$

$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + \frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + C$

$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$

16. $\int (2x - 3\cos x + e^x)dx$

16. $\int (2x - 3xosx + e^x)dx$

$= 2\int xdx - 3\int cosxdx + \int e^x dx$

$= \frac{2x^2}{2} - 3(sinx) + e^x + C$

$= x^2 - 3sinx + e^x + C$

17. $\int \left(2x^2 - 3\sin x + 5\sqrt{x}\right)dx$

17. $\int \left(2x^2 - 3\sin x + 5\sqrt{x}\right)dx$

$= 2\int x^2 dx - 3\int \sin x dx + 5\int x^{\frac{1}{2}}dx$

$= \frac{2x^3}{3} - 3(-\cos x) + 5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{2}{3}x^2 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C$

18. $\int \sec x(\sec x + \tan x)dx$

18. $\int secx(secx + tanx)dx$

$= \int (\sec^2 x + \sec x \tan x)\, dx$

$= \int \sec^2 x\, dx + \int \sec x \tan x\, dx$

$= \tan x + \sec x + C$

19. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

19. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx$

$= \int \frac{\sin^2 x}{\cos^2 x}$

$= \int (\sec^2 x - 1) dx$

$= \int \sec^2 x dx = \int 1 dx$

$= \tan x - x + C$

20. $\int \frac{2 - 3\sin x}{\cos^2 x}$

20. $\int \frac{2-3\sin x}{\cos^2 x} dx$

$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right) dx$

$= \int 2\sec^2 x dx - 3 \int \tan x \sec x dx$

$= 2\tan x - 3\sec x + C$

21. The anti-derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(a) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (b) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$

(c) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (d) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

21. $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

$= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$$

Hence, the correct Answer is C.

22. If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that f(2) = 0, then f(x) is

(a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (b) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

22. It is given that $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$

$$f(x) = \int 4x^3 - \frac{3}{x^4}dx$$

$$\Rightarrow f(x) = 4\int x^3 dx - 3\int (x^{-4})dx \qquad \Rightarrow f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$$

$$\Rightarrow f(x) = x^4 + \frac{1}{x^3} + C$$

Also, It is given that f(2) = 0

$$\Rightarrow f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow C = \frac{-129}{8}$$

Therefore, $f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$