NCERT Solutions for Class-XI Maths

Chapter-2 Exercise-2.1

NCERT Math Class 11

1. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$, find the values of x and y.

1. It is given that

$\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore. $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\frac{x}{3}+1=\frac{5}{3}$

$\Rightarrow \frac{x}{3}=\frac{5}{3}-1 \quad y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{x}{3}=\frac{2}{3} \quad \Rightarrow y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow x=2 \Rightarrow y=1$

$\therefore x=2$ and $y=1$

2. If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A×B).

2. Set A has 3 elements and the set B = {3, 4, 5}

As we see the number of elements in set B = 3.

Number of elements in (A× B) = (Number of elements in set A) × (Number of elements in B)

$\Rightarrow n(A \times B) = n(A) \times n(B)$

$\Rightarrow n(A \times B) = 3 \times 3$

$\Rightarrow n(A \times B) = 9$

Hence, the number of elements in (A×B) = 9.

3. If $G=\{7,8\}$ and $H=\{5,4,2\}$, find $G\times H$ and $H\times G$.

3. $G=\{7,8\}$ and $H=\{5,4,2\}$

We know that the Cartesian product $P \times Q$ of two non-empty sets P and Q is defined as $P \times Q = \{(p,q) : p \in P, q \in Q\}$

$\therefore G \times H = \{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$

$H \times G = \{(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)\}$

4. State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.
(i) If P = {m, n} and Q = { n, m}, then P × Q = {(m, n),(n, m)}.

4. P = {m, n} and Q = { n, m}
By definition of Cartesian product of two non empty Set P and Q:
P × Q = {(p, q): p Є P, q Є Q}
Therefore, P × Q = {(m, n), (m, m), (n, m), (n, n)}.
⇒ P × Q ≠ {(m, n),(n, m)}.
Hence, the statement is false.
(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that x ∈ A and y ∈ B.
Solution: Given: A and B are non-empty sets and x Є A and y Є B.
By definition of Cartesian product of two non empty Set P and Q:
P × Q = {(p, q): p Є P, q Є Q}
➡ A × B = {(x, y): x Є A, y Є B}
Hence, the statement is true.

(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ϕ) = ϕ.
Solution: Given: A = {1, 2}, B = {3, 4}
To Prove: $A \times (B \cap \phi) = \phi$
As $(B \cap \phi) = \phi$
By definition if either of the two set P and Q is null set then P × Q will also be a null set.
i.e. P × Q = ϕ .

⇒ $A \times (B \cap \phi) = \phi$.
Hence,the statement is true.

5. If $A = \{-1,1\}$, find $A \times A \times A$.

5. It is known that for any non-empty set A, $A \times A \times A$ is defined as

$A \times A \times A = \{(a,b,c) : a,b,c \in A\}$

It is given that $A = \{-1,1\}$

$\therefore A\times A\times A=\{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1),$
$(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\}$

6. If A × B = {(a, x),(a , y), (b, x), (b, y)}. Find A and B.

6. A × B = {(a, x),(a , y), (b, x), (b, y)}
By definition of Cartesian product of two non empty Set P and Q:
P × Q = {(p, q): p Є P, q Є Q}
Hence, we see A = set of all first elements.
B = set of all second elements.

⇒ A = {a,b} and B = {x,y}.

7. Let $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ and $D=\{5,6,7,8\}$. Verify that
(i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$
(ii) $A\times C$ is a subset of $B\times D$

7. (i) To verify: $A\times(B\cap C)=(A\times B)\cap(A\times C)$
We have $B\cap C=\{1,2,3,4\}\cap\{5,6\}=\Phi$
$\therefore$ L.H.S. $=A\times(B\cap C)=A\times\Phi=\Phi$
$A\times B=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$
$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$
$\therefore$ R.H.S. $=(A\times B)\cap(A\times C)=\Phi$
$\therefore$ L.H.S. = R.H.S
Hence, $A\times(B\cap C)=(A\times B)\cap(A\times C)$
(ii) To verify: $A\times C$ is a subset of $B\times D$
$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$
$A\times D=\{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),$
$(3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8)\}$
We can observe that all the elements of set $A\times C$ are the elements of set $B\times D$.
Therefore, $A\times C$ is a subset of $B\times D$.

8. Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

8. A = {1, 2} and B = {3, 4}
(A × B) = {(1,3), (1,4), (2,3), (2,4)}
Number of elements in (A × B) = 4

$\Rightarrow$ n(A × B)= 4

Then number of subsets of set (A × B) = $2^n = 2^4 = 16$

$\Rightarrow$ number of subsets of set (A × B) = 16

These are:

{ϕ}, {(1,3)}, {(1,4)}, {(2,3)}, {(2,4)}, {(1,3),(1,4)}, {(1,3),(2,3)}, {(1,3),(2,4)}, {(1,4),(2,3)}, {(1,4),(2,4)}, {(2,3),(2,4)}, {(1,3),(1,4),(2,3)}, {(1,3),(1,4),(2,4)}, {(1,4),(2,3),(2,4)}, {(1,3),(2,3), (2,4)}, {(1,3),(1,4),(2,3),(2,4)}.

9. Let A and B be two sets such that $n(A)=3$ and $n(B)=2$. If $(x,1),(y,2),\ (z,1)$ are in $A\times B$, find A and B, where x,y and z are distinct elements.

9. It is given that $n(A)=3$ and $n(B)=2$; and $(x,1),(y,2),(z,1)$ are in $A\times B$.

We know that

$A=$ Set of first elements of the ordered pair elements of $A\times B$

$B=$ Set of second elements of the ordered pair elements of $A\times B$.

$\therefore x, y$, and z are the elements of A; and 1 and 2 are the elements of B.

Since $n(A)=3$ and $n(B)=2$,

it is clear that $A=\{x,y,z\}$ and $B=\{1,2\}$.

10. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0,1). Find the set A and the remaining elements of A × A.

10. Cartesian product A × A having 9 elements among which are found (–1, 0) and (0,1).

Number of elements in (A× B) = (Number of elements in set A) × (Number of elements in B)

$\Rightarrow$ n(A × A) = n(A) × n(A)

$\Rightarrow$ n(A × A) = 9 (given)

$\Rightarrow$ n(A) × n(A) = 9

$\Rightarrow$ n(A) = 3

By definition A × A = {(a, a): a Є A}.

Therefore, -1, 0 and 1 are the elements of set A.

Because, n(A) = 3 therefore, A = {-1, 0, 1}.

Hence the remaining elements of set (A × A) are:

(-1,-1), (-1,1), (0,0), (0, -1), (1,1), (1, -1) and (1, 0).