NCERT Solutions for Class 10 Maths Chapter 13 Surface Area and Volume

EXERCISE 13.5

1. A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3

Solution: We know that,

No. of rounds = $\frac{Height}{Diameter}$

= $\frac{12}{0.3}$ = 40 rounds

Length of wire required in 1 round = Circumference of base of cylinder

= $2\pi r$

= $2\pi \times 5$

= 10π

Length of wire in 40 rounds = $40 \times 10\pi$

= $\frac{400*22}{7}$

= $\frac{8800}{7}$ = 12.57 m

Radius of wire = $\frac{0.3}{2}$ = 0.15 m

Volume of wire = Area of cross-section of wire × Length of wire

= $\pi(0.15)^2 \times 1257.14$

= 88.898 cm^3

Mass = Volume × Density

$= 88.898 \times 8.88$

$= 789.41$ gm

2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate)

Solution: As per the question,

Hypotenuse, AC = $\sqrt{3 * 3 + 4 * 4}$

= 5 cm

Area of ΔABC = $\frac{1}{2}$ * AB * AC

$\frac{1}{2}$ * AC * OB = $\frac{1}{2}$ * 4 * 3

OB = $\frac{12}{5}$ = 2.4 cm

Volume of double cone = Volume of cone 1 + Volume of cone 2

$= \frac{1}{3}\pi r^2 h1 + \frac{1}{2}\pi r^2 h2$

$= \frac{1}{3}\pi r^2 (h1 + h2)$

$= \frac{1}{3}\pi r^2 (OA + OC)$

$= \frac{1}{3} * 3.14 * (2.4)^2 (5)$

$= 30.14$ cm^3

Surface area of double cone = Surface area of cone 1 + Surface area of cone 2

$= \pi r l1 + \pi r l2$

$= \pi r [4 + 3]$

$= 3.14 * 2.4 * 7$

$= 52.75\ cm^2$

3. A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Solution: According to the question:

Volume of cistern = 150 × 120 × 110

$= 1980000\ cm^3$

Volume to be filled in cistern = 1980000 - 129600

$= 1850400\ cm^3$

Let n numbers of porous bricks were placed in the cistern

Volume of n bricks = $n \times 22.5 \times 7.5 \times 6.5$

$= 1096.875n$

As each brick absorbs one-seventeenth of its volume, therefore, volume absorbed by these bricks = $\frac{n}{17}(1096.875)$

$1850400 + \frac{n}{17}(1096.875) = (1096.875)n$

$1850400 = \frac{16n}{17}(1096.875)$

n = 1792.41

4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is $97280\ km^2$, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep

Solution: Area of the valley = 97280 km^2

If there was a rainfall of 10 cm in the valley then amount of rainfall in the valley = area of the valley × 10 cm

Amount of rainfall in the valley = 97280 km^2× 10 cm

Amount of rainfall in one day=9.728×10^9 * 14

=0.694×10^9

=6.94×10^8 m^3

Length of each river, l = 1072 km

= 1072 × 1000 m

= 1072000 m

Breadth of each river, b = 75 m

Depth of each river, h = 3 m

Volume of each river = $l \times b \times h$

= 1072000 × 75 × 3 m^3

= 2.412×10^8 m^3

Volume of three such rivers = 3 × Volume of each river

= $3 \times 2.412 \times 10^8$ m^3

= 7.236×10^8 m^3

5. An oil funnel made of tin sheet consists of a10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 semidiameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see Fig. 13.25)

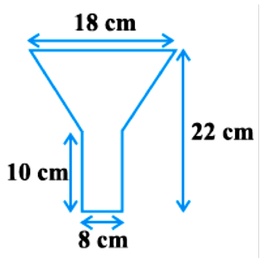


Fig. 13.25

Solution: Given:

Radius ($r1$) of upper circular end of frustum part = $\frac{18}{2}$ = 9 cm

Radius ($r2$) of lower circular end of frustum part = Radius of circular end of cylindrical part = $\frac{8}{2}$ = 4 cm

Height ($h1$) of frustum part = 22 - 10 = 12 cm

Height ($h2$) of cylindrical part = 10 cm

Slant height (l) of frustum part

= $\sqrt{(9-4) * (9-4) + (12) * 12}$

= 13 cm

Area of tin sheet required = CSA of frustum part + CSA of cylindrical part

= π (r1 + r2) * l + 2πr2h2

= $\frac{22}{7}$ (9 + 4) * 13 + 2 * $\frac{22}{7}$ * 4 * 10

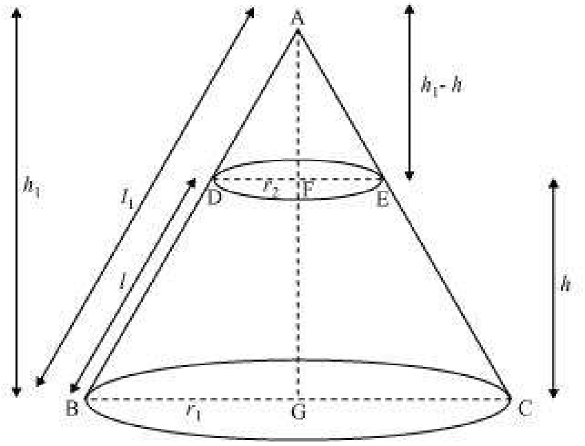
= $\frac{22}{7}$ [169 + 80]

= $\frac{22*249}{7}$

= 782 $\frac{4}{7}$ cm^2

6. Derive the formula for the curved surface area and total surface area of the frustum of acone, given to you in Section 13.5, using the symbols as explained

Solution: Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base. Let $r1$ and $r2$ be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone



In ΔABG and ΔADF, DF||BG

$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$

$\frac{r2}{r1} = \frac{h1-h}{h1} = \frac{l1-l}{l1}$

$\frac{r2}{r1} = 1 - \frac{h}{h1} = 1 - \frac{l}{l1}$

$1 - \frac{l}{l1} = \frac{r2}{r1}$

$\frac{l}{l1} = 1 - \frac{r2}{r1} = \frac{r1-r2}{r1}$

$\frac{l1}{l} = \frac{r1}{r1-r2}$

$l1 = \frac{r1l}{r1-r2}$

CSA of frustum DECB = CSA of cone ABC - CSA cone ADE

= πr1l1 – πr2 (l1 - l)

$= \pi r1 \left(\frac{lr1}{r1-r2}\right) - \pi r2 \left[\frac{r1l}{r1-r2} - l\right]$

$= \frac{\pi * r1 * r1 * l}{r1-r2} - \frac{\pi * r2 * r2 * l}{r1-r2}$

$= \pi l \left[\frac{r1*r1-r2*r2}{r1-r2} \right]$

CSA = π (r1 + r2) * l

Total surface area of frustum = CSA + Area of upper circular end + Area of lower circular end

$= \pi (r1 + r2) * l + \pi r2^2 + \pi r1^2$

$= \pi [(r1 + r2) * l + r1^2 + r2^2]$

7. Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5,using the symbols as explained

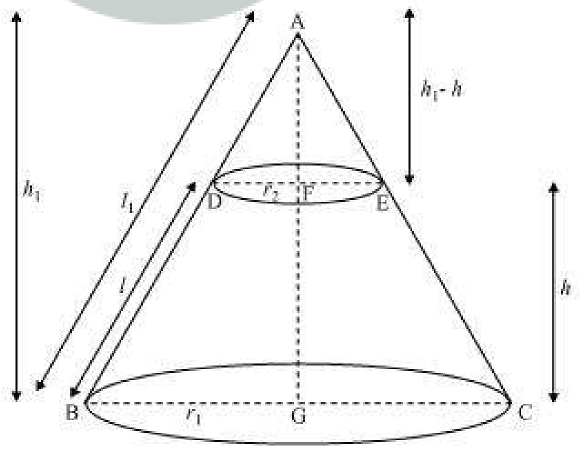
Solution: Let ABC be a cone.

And,

A frustum DECB is cut by a plane parallel to its base

Now,

Let *r*1 and *r*2 be the radii of the ends of the frustum of the cone and *h* be the height of the frustum of the cone



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$\frac{h1}{h} = \frac{r1}{r1-r2}$

$h1 = \frac{r1h}{r1-r2}$

Volume of frustum of cone = Volume of cone ABC - Volume of cone ADE

$= \frac{1}{3}\pi r1^2 h1 - \frac{1}{3}\pi r2^2 (h1 - h)$

$= \frac{\pi}{3} [r1^2 h1 - r2^2 (h1 - h)]$

$= \frac{\pi}{3} [r1^2 (\frac{hr1}{r1-r2}) - r2^2 (\frac{hr1}{r1-r2} - h)]$

$= \frac{\pi}{3} [\frac{h*r1*r1*r1}{r1-r2} - \frac{h*r2*r2*r2}{r1-r2}]$

$= \frac{\pi}{3} * h [\frac{r1*r1*r1-r2*r2*r2}{r1-r2}]$

$= \frac{\pi}{3} h [\frac{(r1-r2)(r1*r1+r2*r2+r1*r2)}{r1-r2}]$

$= \frac{1}{3}\pi h [r1^2 + r2^2 + r1r2]$