NCERT Solutions for Class-XII Physics

Chapter-13

NCERT Physics Class 12

1. (a) Two stable isotopes of lithium ${}_{3}^{6}\text{Li}$ and ${}_{3}^{7}\text{Li}$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, ${}_{5}^{10}\text{B}$ and ${}_{5}^{11}\text{B}$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of ${}_{5}^{10}\text{B}$ and ${}_{5}^{11}\text{B}$.

1. (a) Mass of ${}_{3}^{6}\text{Li}$ lithium isotope, m_1 = 6.01512 u

Mass of ${}_{3}^{7}\text{Li}$ lithium isotope, m_2 = 7.01600 u

Abundance of ${}_{3}^{6}\text{Li}, \eta_1 = 7.5\%$

Abundance of ${}_{3}^{7}\text{Li}, \eta_2 = 92.5\%$

The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2} = \frac{6.01512 \times 7.5 \times 7.01600 \times 92.5}{7.5 + 92.5} = 6.940934 \text{ u}$$

(b) Mass of ${}_{5}^{10}\text{B}$ boron isotope, m_1 = 10.01294 u

Mass of ${}_{5}^{11}\text{B}$ boron isotope, m_2 = 11.00931 u

Let the abundance of ${}_{5}^{10}\text{B}, \eta_1 = x\ \%$

Therefore, the abundance of ${}_{5}^{11}\text{B}, \eta_2 = (100 - x)\%$

Atomic mass of boron = 10.811 u. The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$\Rightarrow 10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + (100 - x)}$$

$\Rightarrow$ 1081.1 = 10.01294x + 1100.931 + 1100.931x

$$\Rightarrow x = \frac{19.821}{0.99637} = 19.89\%$$

Therefore, 100 – x = 100 % – 19.89 % = 80.11 %

Hence, the abundances of ${}_{5}^{10}\text{B}$ is 19.89 % and ${}_{5}^{11}\text{B}$ is 80.11 %.

2. The three stable isotopes of neon: ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27%, and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

2. The atomic mass and abundance of ${}^{20}Ne_{10}$ is m_1 = 19.99 u, n_1 = 90.51%
The atomic mass and abundance of ${}^{21}Ne_{10}$ is m_2 = 20.99 u, n_2 = 0.27%
The atomic mass and abundance of ${}^{22}Ne_{10}$ is m_3 = 21.99 u ,n_3 = 9.22%

$$\text{Mass} = \frac{\text{Mass of 1st isotope} \times \text{it's abundance} + \text{mass of 2nd isotope} \times \text{it's abundance}}{\sum \text{of both the isotope's abundances}}$$

Hence average atomic mass of Neon $= \dfrac{m_1n_1 + m_2n_2 + m_3n_3}{n_1 + n_2 + n_3}$

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{100} \text{u}$$

= 20.1771 u
Average atomic mass of Neon = 20.1771 u.

3. Obtain the binding energy (in MeV) of a nitrogen nucleus ${}^{14}_{7}N$, given $m\left({}^{14}_{7}N\right)$ = 14.00307 u.

3. Atomic mass of ${}^{14}_{7}N$ nitrogen, m = 14.00307 u
A nucleus of ${}^{14}_{7}N$ nitrogen contains 7 protons and 7 neutrons.
Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$
Where,
Mass of proton, m_H = 1.007825 u
Mass of a neutron, m_n = 1.008665 u
$\therefore \Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307 = 7.054775 + 7.06055 - 14.00307 =$ 0.11236 u
But 1 u = 931.5 MeV/c^2
$\therefore \Delta m = 0.11236 \times 931.5$ MeV/c^2
Hence, the binding energy of the nucleus is given as:
$E_b = \Delta mc^2$ Where, c = Speed of light

$$\therefore E_b = 0.11236 \times 931.5\left(\frac{\text{MeV}}{c^2}\right)c^2 = 104.66334 \text{ MeV}$$

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

4. Obtain the binding energy of the nuclei ${}^{56}_{26}Fe$ and ${}^{209}_{83}Bi$ in units of MeV from the following data:

$m\left({}^{56}_{26}Fe\right) = 55.934939$ u, $m\left({}^{209}_{83}Bi\right) = 208.980388$ u

4. The number of neutron and Proton both in a ${}^{56}Fe_{26}$ atom is respectively 30 and 26.
The mass defect $\Delta m = m_p + m_n - m_{Fe}$
And Binding energy is given by $E_B = \Delta mc^2$
Mass of a proton = 1.007825 u
Mass of a neutron = 1.008665 u
In this case mass defect = $\Delta m = (26 \times 1.007825 + 30 \times 1.008665 - 55.934939)u = 0.528461$ u
We know, 1 u = 931.5 MeV/c^2
So, $E = \Delta mc^2 = 0.528461 \times 931.5(MeV/c^2) \times c^2 = 492.26MeV$
So, Binding Energy per Nucleon $= \dfrac{\text{Total Binding Energy}}{\text{Total No of Nucleon}} = \dfrac{492.26}{56} = 8.79$ MeV
The number of neutron and Proton both in a ${}^{209}Bi_{83}$ atom is respectively 126 and 83.
The mass defect $\Delta m = m_p + m_n - m_{Bi}$
And Binding energy is given by $E_B = \Delta mc^2$
Mass of a proton = 1.007825 u
Mass of a neutron = 1.008665 u
Mass Defect = $\Delta m = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388 = 1.760877$ u
We know, 1 u = 931.5 MeV/c^2
So, $E = \Delta mc^2 = 1.760877 \times 931.5\ (Mev/c^2) \times c^2 = 1640.26$ MeV
Average binding energy per nucleon = 1640.26/209 = 7.848 MeV

5. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}^{63}_{29}Cu$ atoms (of mass 62.92960 u).

5. Mass of a copper coin m' = 3 g
Atomic mass of ${}^{69}_{29}Cu$ atom, m = 62.92960 u
The total number of ${}^{63}_{29}Cu$ atoms in the coin, $N = \dfrac{N_A \times m'}{\text{Mass Number}}$
Where,
N_A = Avogadro's number = 6.023×10^{23} atoms/g and mass number = 63 g
$$N = \frac{6.022 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22} \text{ atoms}$$
${}^{63}_{29}Cu$ nucleus has 29 protons and (63 – 29) 34 neutrons.
∴ Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$
Where, Mass of a proton, $m_H = 1.007825$ u
Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296 = 0.591935$ u

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22} = 1.69766958 \times 10^{22}$ u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5$ MeV/c^2

Hence, the binding energy of the nuclei of the coin is given as:

$$E_b = \Delta mc^2 = 1.69766958 \times 10^{22} \times 931.5\left(\frac{\text{MeV}}{c^2}\right)c^2 = 1.581 \times 10^{25} \text{ MeV}$$

But 1 MeV = 1.6×10^{-13} J

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13} = 2.5296 \times 10^{12}$ J

This much energy is required to separate all the neutrons and protons from the given coin.

6. Write nuclear reaction equations for

(i) α -decay of $^{226}_{88}\text{Ra}$

(ii) α -decay of $^{242}_{94}\text{Pu}$

(iii) β– -decay of $^{32}_{15}\text{P}$

(iv) β– -decay of $^{210}_{83}\text{BI}$

(v) β+ -decay of $^{12}_{6}\text{C}$

(vi) β+ -decay of $^{97}_{43}\text{Tc}$

(vii) Electron capture of $^{120}_{54}\text{Xe}$

6. Explanation for the below-given solution in a nutshell, In case of α decay , there is a loss of 2 protons And 4 neutrons , In case of β^+ decay there is a loss of a proton and a neutrino is emitted, and for every β^- decay there is a gain of one proton and an antineutrino is emitted.

Hence the equations for the given cases will be:

(i) $\text{Ra}^{226}_{88} \rightarrow \text{Rn}^{222}_{86} + \text{He}^4_2\ (\alpha - \text{decay})$

(ii) $\text{Pu}^{242}_{94} \rightarrow \text{U}^{238}_{92} + \text{He}^4_2\ (\alpha - \text{decay})$

(iii) $\text{P}^{32}_{15} \rightarrow \text{S}^{32}_{16} + e^- + \bar{v}(\beta - \text{decay})$

(iv) $\text{B}^{210}_{83} \rightarrow \text{Po}^{210}_{84} + e^- + \bar{v}(\beta^- - \text{decay})$

(v) $\text{C}^{11}_{6} \rightarrow \text{B}^{11}_{5} + e^+ + v(\beta^+ - \text{decay})$

(vi) $\text{Tc}^{97}_{43} \rightarrow \text{Mo}^{97}_{42} + e^+ + v(\beta^+ - \text{decay})$

(vii) $\text{Xe}^{120}_{54} + e^+ \rightarrow \text{I}^{120}_{53} + v(\text{electron capture})$

7. A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to (a) 3.125%, b) 1% of its original value?

7. Half-life of the radioactive isotope = T years and original amount of the radioactive isotope = N_0

(a) After decay, the amount of the radioactive isotope = N

It is given that only 3.125 % of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

But $\frac{N}{N_0} = e^{-\lambda t}$

Where, λ = Decay constant and t = Time

$$\therefore -\lambda t = \frac{1}{32}$$

$-\lambda t = \ln 1 - \ln 32$

$-\lambda t = 0 - 3.4657$

$$t = \frac{3.4657}{\lambda}$$

Since $\lambda = \frac{0.693}{T}$

$$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5T \text{ years}$$

Hence, the isotope will take about 5T years to reduce to 3.125 % of its original value.

(b) After decay, the amount of the radioactive isotope = N

It is given that only 1% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

But $\frac{N}{N_0} = e^{-\lambda t}$

$$\therefore e^{-\lambda t} = \frac{1}{100}$$

$-\lambda t = \ln 1 - \ln 100$

$-\lambda t = 0 - 4.6052$

$$t = \frac{4.6052}{\lambda}$$

Since, $\lambda = 0.693/T$

$$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645\ T \text{ years}$$

Hence, the isotope will take about 6.645T years to reduce to 1% of its original value.

8. The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilization.

8. Let N be a number of radioactive carbon found in normal carbon and N_0 be the number of radioactive carbon found in the specimen. The half-life of C-14 is 5730 yrs.
Decay rate of living carbon-containing matter = D = 15 decay/min-gm
Decay rate of the specimen at Mohenjo-Daro = D_0 = 9 decays/min-gm
From the exponential decay rate law, we get,

$\therefore$ Hence, $\frac{N}{N_0} = \frac{D}{D_0} = e^{-\lambda t} = \frac{9}{15}$

So, $-\lambda t = \ln(3/5) = -0.5108$

$t = \frac{0.5108}{\lambda} = \frac{0.5108}{0.693/T_{\frac{1}{2}}}$

or, $t = \frac{0.5108 \times 5730}{0.693} = 4223.5$ years

Approximate age of Indus-Valley-Civilization is 4223.5 years.

9. Obtain the amount of ${}^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of ${}^{60}_{27}Co$ is 5.3 years.

9. The strength of the radioactive source is given as:

$\frac{dN}{dt} = 8.0 \text{ mCi}$

$= 8 \times 10^{-3} \times 3.7 \times 10^{10}$

$= 29.6 \times 10^{7}$ decay/s

Where, N = Required number of atoms

Half-life of ${}^{60}_{27}Co$, $T_{1/2}$ = 5.3 years = $5.3 \times 365 \times 24 \times 60 \times 60 = 1.67 \times 10^{8}$ s

For decay constant λ, we have the rate of decay as:

$\frac{dN}{dt} = \lambda N$

Where, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^8} s^{-1}$

$\therefore N = \frac{1}{\lambda}\frac{dN}{dt}$

$= \frac{29.6 \times 10^7}{\frac{0.693}{1.67 \times 10^8}} = 7.133 \times 10^{16}$ atoms

For $^{60}_{27}Co$, mass of 6.023×10^{23} (Avogadro's number) atoms = 60 g

$\therefore$ Mass of 7.133×10^{16} atoms $= \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6}$ g

Hence, the amount of $^{60}_{27}Co$ necessary for the purpose is 7.106×10^{-6} g.

10. The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

10. From the principle of exponential radioactive decay,

we know that $\frac{dN}{dt} = \lambda N$

Here, the mass of the isotope is = 15 g

No. of atoms in 15gm of atom is = $\frac{6.023 \times 10^{23} \times 15 \times 10^{-3}}{90} = 1.0038 \times 10^{20}$ atoms

And, $\lambda = (0.693/T_{1/2})$ where, Half life = 28 × 365 × 86400 = 8.83 × 10^8 secs

Hence, $\frac{dN}{dt} = \frac{0.693 \times 1.0038 \times 10^{20}}{8.83 \times 10^8} = 7.878 \times 10^{10}$ atoms / sec

The disintegration rate of 15 mg of the isotope will be 7.878×10^{10} atoms/sec.

11. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

11. Nuclear radius of the gold isotope $^{197}_{79}Au = R_{Au}$

Nucleus radius of the silver isotope $^{107}_{47}Ag = R_{Ag}$

Mass number of gold, A_{Au} = 197

Mass number of silver, A_{Ag} = 107

The ratio of the radii of the two nuclei is related with their mass number as:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{R_{Aa}}{R_{Ag}}\right)^{\frac{1}{3}}$$

$$=\left(\frac{197}{107}\right)^{\frac{1}{3}}=1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

12. Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of (a) ${}^{226}_{88}\mathrm{Ra}$ and (b) ${}^{220}_{86}\mathrm{Rn}$.

Given: $m\left({}^{226}_{88}\mathrm{Ra}\right)=226.02540$ u, $m\left({}^{222}_{86}\mathrm{Rn}\right)=222.01750$ u,

$m\left({}^{220}_{86}\mathrm{Rn}\right)=220.01137$ u, $m\left({}^{216}_{84}\mathrm{Po}\right)=216.00189$ u.

12. (a) Q value of emitted α particle = (Total Initial mass – Total final mass) × c^2

∴ The α particle decay of ${}^{226}\mathrm{Ra}_{88}$ is given by:

$${}^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$$

Hence the Q value will be = [226.02540 – (222.01750 + 4.002603)] × c^2

= 0.005297 u × c^2

= 0.005297 × 931.5 Mev

= 4.94 MeV

Kinetic energy is given by $=\frac{\text{mass after decay}}{\text{mass before decay}}\times Q_{\text{value}}$

$=\frac{222}{226}\times 4.94$ Mev

= 4.85 MeV

(b) Similarly, the decay of ${}^{220}\mathrm{Rn}_{86}$ is given by:

$${}^{220}_{86}\mathrm{Rn} \rightarrow {}^{216}_{84}\mathrm{Po} + {}^{4}_{2}\mathrm{He}$$

Hence, Q value = [220.01137 – (216.00189 + 4.0026)]u × c^2 = 0.00688u × 931.5 MeV

= 6.41MeV

Kinetic energy = $\frac{\text{mass afer decay}}{\text{mass before decay}}\times Q$ value

$=6.41\times\frac{216}{220}$ MeV

= 6.293 MeV

13. The radionuclide ${}^{11}\mathrm{C}$ decays according to

$${}^{11}_{6}\mathrm{C} \rightarrow {}^{11}_{5}\mathrm{B} + e^{+} + v: \quad T_{1/2}=20.3\ \mathrm{min}$$

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values:

$m\left({}^{11}_{6}\mathrm{C}\right)=11.011434$ u and $m\left({}^{11}_{6}\mathrm{B}\right)=11.009305$ u,

calculate Q and compare it with the maximum energy of the positron emitted.

13. The given values are:

$m\left({}^{11}_{6}C\right) = 11.011434\ u$ and $m\left({}^{11}_{6}B\right) = 11.009305\ u$

The given nuclear reaction is:

${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + v$

Half-line of ${}^{11}_{6}C$ nuclei, $T_{1/2}$ = 20.3 min

Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the Q-value (ΔQ) of the nuclear masses of the ${}^{11}_{6}C$

$$\Delta Q = \left[m'\left({}^{11}_{6}C\right) - \left\{m'\left({}^{11}_{5}B\right) + m_e\right\}\right]c^2$$

Where, m_e = Mass of an electron or positron = 0.000548 u

c = Speed of light and m' = Respective nuclear masses.

If atomic masses are used instead of nuclear masses, then we have to add 6 m_e in the case of ${}^{11}_{6}C$ and

5 m_e in the case of ${}^{11}_{5}B$.

Hence, equation (1) reduces to:

$$\Delta Q = \left[m\left({}^{11}_{6}C\right) - m\left({}^{11}_{5}B\right) - 2m_e\right]c^2$$

Here, $\left({}^{11}_{6}C\right)$ and $\left({}^{11}_{5}B\right)$ are the atomic masses.

∴ $\Delta Q = [11.011434 - 11.009305 - 2 \times 0.000548]c^2 = (0.001033\ c^2)u$

But 1 u = 931.5 Mev/c^2

$\Delta Q = 0.001033 \times 931.5 \approx 0.962$ MeV

The value of Q is almost comparable to the maximum energy of the emitted positron.

14. The nucleus ${}^{23}_{10}Ne$ decays by β^- emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m\left({}^{23}_{10}Ne\right) = 22.994466\ u$

$m\left({}^{23}_{11}Na\right) = 22.089770\ u$

14. For β^- decay,the number of proton increases by 1.The reaction is given as:

$Ne^{23}_{10} \rightarrow Na^{23}_{11} + e^{-} + \overline{v} + Q$

Here the electron masses gets cancelled as Na has one more electron than Ne hence,

Q = (22.994466 – 22.089770)u × c^2

= 0.00469 × 931.5 MeV

= 4.37 MeV

The maximum k.E of the emitted electrons are comparable to the Q value; Hence the maximum K.E of the electrons emitted will be = 4.37 MeV

15. The Q value of a nuclear reaction A + b → C + d is defined by

$Q = [m_A + m_b - m_C - m_d]c^2$, where the masses refer to the respective nuclei.

Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}^1_1H + {}^3_1H \to {}^2_1H + {}^2_1H$

(ii) ${}^{12}_6C + {}^{12}_6C \to {}^{20}_{10}Ne + {}^4_2He$

Atomic masses are given to be

$m\left({}^2_1H\right) = 2.014102$ u

$m\left({}^3_1H\right) = 3.016049$ u

$m\left({}^{12}_6C\right) = 12.000000$ u

$m\left({}^{20}_{10}Ne\right) = 19.992439$ u

15. (i) The given nuclear reaction is:

It is given that: ${}_1H^1 + {}^3_1H \to {}^2_1H + {}^2_1H$

Atomic mass $m\left({}^1_1H\right) = 1.007825$ u

Atomic mass $m\left({}^3_1H\right) = 3.016049$ u

Atomic mass $m\left({}^2_1H\right) = 2.014102$ u

According to the question, the Q-value of the reaction can be written as:

$$Q = \left[m\left({}^1_1H\right) + m\left({}^3_1H\right) - 2m\left({}^2_1H\right)\right]c^2$$

$$= \left[1.007825 + 3.016049 - 2 \times 2.014102\right]c^2$$

Q = $(-0.00433\ c^2)$ u

But 1 u = 931.5 MeV/c^2

∴ Q = –0.00433 × 931.5 = –4.0334 MeV

The negative Q-value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:

It is given that: ${}^{12}_6C + {}^{12}_6C \to {}^{20}_{10}Ne + {}^4_3He$

Atomic mass of $m\left({}^{12}_6C\right) = 12.0$ u

Atomic mass of $m\left({}^{20}_{10}Ne\right) = 19.992439$ u

Atomic mass of $m\left({}^4_2He\right) = 4.002603$ u

The Q-value of this reaction is given as:

$$Q = \left[2m\left({}^{12}_6C\right) - m\left({}^{20}_{10}Ne\right) - m\left({}^4_2He\right)\right]c^2$$

$= [2 \times 12.0 - 19.992439 - 4.002603]c^2$
$= (0.004958\, c^2)u$
$= 0.004958 \times 931.5 = 4.618377$ MeV
The positive Q-value of the reaction shows that the reaction is exothermic.

16. Suppose, we think of fission of a ${}^{56}_{26}Fe$ nucleus into two equal fragments, ${}^{28}_{13}Al$. Is the fission energetically possible? Argue by working out Q of the process. Given $m({}^{56}_{26}Fe)$ = 55.93494 u and $m({}^{28}_{13}Al)$ = 27.98191 u.

16. The fission reaction can be given as: ${}^{56}_{26}Fe \rightarrow 2\,{}^{28}_{13}Al$

The Q value for this reaction will be given as = $\left[m({}^{56}_{26}Fe) - 2 \times m({}^{28}_{13}Al)\right] \times c^2$

$= (55.93494 - 2 \times 27.98191)\, u \times c^2$ MeV
$= -\,0.02888u \times c^2$ MeV
$= -\,0.02888 \times 931.5$
$= -\,26.902$ MeV
The Q value is negative which suggests this reaction is endothermic, but we know fission reactions are exothermic. Hence, this fission reaction is not energetically possible.

17. The fission properties of ${}^{239}_{94}Pu$ are very similar to those of ${}^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}^{239}_{94}Pu$ undergo fission?

17. Average energy released per fission of ${}^{239}_{94}94Pu, E_{av} = 180\, MeV$

Amount of pure ${}^{239}_{94}Pu$, m = 1 kg = 1000 g

N_A = Avogadro number = 6.023×10^{23}

Mass number of ${}^{239}_{94}Pu = 239\, g$

1 mole of ${}^{239}_{94}Pu$ contains N_A atoms.

$\therefore$ mg of ${}^{239}_{94}Pu$ contains $\left(\frac{N_A}{\text{Mass number}} \times m\right)$ atoms

$\therefore$ 1000 g of ${}^{239}_{94}Pu$ contains $\left(\frac{N_A}{\text{Mass number}} \times 1000\right)$ atoms $= 2.52 \times 10^{24}$ atoms

$\therefore$ Total energy released during the fission of 1 kg of ${}^{239}_{94}Pu$ is calculated as:

$E = E_{av} \times 2.52 \times 10^{24}$
$= 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26}$ MeV

Hence, 4.536 × 10^{26} MeV is released if all the atoms in 1 kg of pure ${}^{239}_{94}\text{Pu}$ undergo fission.

18. A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much ${}^{235}_{92}\text{U}$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission

of ${}^{235}_{92}\text{U}$ and that this nuclide is consumed only by the fission process.

18. Half-life of the fuel in the fission reactor = 5 years

= 5 × 365 × 86400 s (∵ 1 day = 86400 seconds)

= 1.576 × 10^8 sec

∵ 1 gm of Uranium fission gives 200 MeV energy [This can be worked out from the fission equation given below of Uranium].

$$\boxed{{}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{90}_{36}\text{Kr} + {}^{143}_{56}\text{Ba} + {}^{1}_{0}\text{n} + {}^{1}_{0}\text{n} + {}^{1}_{0}\text{n}}$$

∴ 1 gm of Uranium, $= \dfrac{6.023 \times 10^{23}}{235}$

Energy generated by Uranium fission $= \dfrac{6.023 \times 10^{23}}{235} \times 200\ \text{MeV}$

$= \dfrac{6.023 \times 10^{23}}{235} \times 200 \times 1.6 \times 10^{-19}\ \text{J}$

= 8.2 × 10^{10} J/gm

∵ The 1000 MW reactor operates only 80% of it's time, hence in 5 years amount of uranium consumed

$= \dfrac{80 \times 1.576 \times 10^8 \times 1000 \times 10^6}{8.2 \times 10^{10}}$

= 1538 kg

∴ This amount is consumed within the half life time.

Hence, the initial amount will be = 2 × 1538 Kg

= 3076 Kg

19. How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{1}\text{He} + \text{n} + 3.27\ \text{MeV}$

19. The given fusion reaction is:

${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + \text{n} + 3.27\ \text{MeV}$

Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains 6.023 × 10^{23} atoms.

$\therefore$ 2.0 kg of deuterium contains $\frac{6.023\times10^{23}}{2}\times2000=6.023\times10^{26}$ atoms

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

$\therefore$ Total energy per nucleus released in the fusion reaction:

$$E=\frac{3.27}{2}\times6.023\times10^{26}\,\text{MeV}$$

$$=\frac{3.27}{2}\times6.023\times10^{26}\times1.6\times10^{-19}\times10^{6}$$

$= 1.576 \times 10^{14}$ J

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$$\frac{1.576\times10^{14}}{100}\text{S}=\frac{1.576\times10^{14}}{100\times60\times60\times24\times365}\approx4.9\times10^{4}\ \text{years}$$

20. Calculate the height of the potential barrier for a head-on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

20. In case of collision of two deuterons, the distance between their centres d is given as

$\because$ d = Radius of first atom + Radius of 2nd atom

radius of deuteron = 2 fm = 2×10^{-15} m

Hence, d = $2 \times 2 \times 10^{-15} = 4 \times 10^{-15}$ m

$\because$ Potential energy of this two deuteron system will be $V=\frac{e^2}{4\pi\epsilon_0 d}$

where ϵ_0 = permittivity of free space

And $\frac{1}{4\pi\epsilon_0}=9\times10^{9}\ \text{m}^2\text{C}^{-2}$

$$V=\frac{9\times10^{9}\times\left(1.6\times10^{-19}\right)^2}{4\times10^{-15}}\text{J}$$

$$\therefore V=\frac{9\times10^{9}\times\left(1.6\times10^{-19}\right)^2}{4\times10^{-15}\times\left(1.6\times10^{-19}\right)}\text{eV}$$

$\therefore$ V = 360 keV

Hence, the height of barrier potential is 360 keV.

21. From the relation $R = R_0A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e., independent of A).

21. We have the expression for nuclear radius as:
$R = R_0 A^{1/3}$
Where, R_0 = Constant
A = Mass number of the nucleus

Nuclear matter density, $\rho = \frac{\text{Mass of the nuclues}}{\text{Volume of the nucleus}}$

Let m be the average mass of the nucleus. Hence, mass of the nucleus = mA

$$\therefore \rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Hence, the nuclear matter density is independent of A. It is nearly constant.

22. For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K–shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}_Z^A X \rightarrow {}_{Z-1}^A Y + v$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice–versa.

22. The reaction for the electron capture process is given by:

$$e^+ + X_Z^A \rightarrow A_{Z-1}^A + v + Q_A$$

The reaction for the β^+ emission is given by:

$$X_Z^A \rightarrow Y_{Z-1}^A + e^+ + v + Q_B$$

Considering the atomic mass of the nucleus we get the following Q value equations.

$$Q_A = m\left(X_Z^A\right) - Zm_e + m_e - m\left(Y_{Z-1}^A\right) + (Z-1)m_e$$

$$= m\left(X_Z^A\right) - m\left(Y_{Z-1}^A\right)$$

And similarly,

$$Q_B = m\left(X_Z^A\right) - zm_e - m\left(Y_{Z-1}^A\right) + (z-1)m_e - m_e$$

$$= m\left(X_Z^A\right) - m\left(Y_{Z-1}^A\right) - 2m_e$$

Here from these two equations we can see if $Q_B > 0$, then $Q_A > 0$ But, If $Q_A > 0$ that doesn't imply
$Q_B > 0$.
Which means, if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice–versa.