

EXERCISE 23.6

1. Find the equation to the straight line

(i) cutting off intercepts 3 and 2 from the axes.

(ii) cutting off intercepts -5 and 6 from the axes.

Solution:

(i) Cutting off intercepts 3 and 2 from the axes.

Given:

$$a = 3, b = 2$$

Let us find the equation of line cutoff intercepts from the axes.

By using the formula,

$$\text{The equation of the line is } x/a + y/b = 1$$

$$x/3 + y/2 = 1$$

By taking LCM,

$$2x + 3y = 6$$

∴ The equation of line cut off intercepts 3 and 2 from the axes is $2x + 3y = 6$

(ii) Cutting off intercepts -5 and 6 from the axes.

Given:

$$a = -5, b = 6$$

Let us find the equation of line cutoff intercepts from the axes.

By using the formula,

$$\text{The equation of the line is } x/a + y/b = 1$$

$$x/-5 + y/6 = 1$$

By taking LCM,

$$6x - 5y = -30$$

∴ The equation of line cut off intercepts 3 and 2 from the axes is $6x - 5y = -30$

2. Find the equation of the straight line which passes through (1, -2) and cuts off equal intercepts on the axes.

Solution:

Given:

A line passing through (1, -2)

Let us assume, the equation of the line cutting equal intercepts at coordinates of length 'a' is

By using the formula,

$$\text{The equation of the line is } x/a + y/b = 1$$

$$x/a + y/a = 1$$

$$x + y = a$$

The line $x + y = a$ passes through $(1, -2)$

Hence, the point satisfies the equation.

$$1 - 2 = a$$

$$a = -1$$

\therefore The equation of the line is $x + y = -1$

3. Find the equation to the straight line which passes through the point $(5, 6)$ and has intercepts on the axes

(i) Equal in magnitude and both positive

(ii) Equal in magnitude but opposite in sign

Solution:

(i) Equal in magnitude and both positive

Given:

$$a = b$$

Let us find the equation of line cutoff intercepts from the axes.

By using the formula,

The equation of the line is $x/a + y/b = 1$

$$x/a + y/a = 1$$

$$x + y = a$$

The line passes through the point $(5, 6)$

Hence, the equation satisfies the points.

$$5 + 6 = a$$

$$a = 11$$

\therefore The equation of the line is $x + y = 11$

(ii) Equal in magnitude but opposite in sign

Given:

$$b = -a$$

Let us find the equation of line cutoff intercepts from the axes.

By using the formula,

The equation of the line is $x/a + y/b = 1$

$$x/a + y/-a = 1$$

$$x - y = a$$

The line passes through the point $(5, 6)$

Hence, the equation satisfies the points.

$$5 - 6 = a$$

$$a = -1$$

\therefore The equation of the line is $x - y = -1$

4. For what values of a and b the intercepts cut off on the coordinate axes by the line $ax + by + 8 = 0$ are equal in length but opposite in signs to those cut off by the line $2x - 3y + 6 = 0$ on the axes.

Solution:

Given:

Intercepts cut off on the coordinate axes by the line $ax + by + 8 = 0$ (i)

And are equal in length but opposite in sign to those cut off by the line

$2x - 3y + 6 = 0$ (ii)

We know that, the slope of two lines is equal

The slope of the line (i) is $-a/b$

The slope of the line (ii) is $2/3$

So let us equate,

$$-a/b = 2/3$$

$$a = -2b/3$$

The length of the perpendicular from the origin to the line (i) is

By using the formula,

$$d = \frac{|ax+by+d|}{\sqrt{a^2+b^2}}$$

$$d_1 = \frac{|a(0)+b(0)+8|}{\sqrt{a^2+b^2}}$$
$$= \frac{8 \times 3}{\sqrt{13b^2}}$$

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The length of the perpendicular from the origin to the line (ii) is

By using the formula,

$$d = \frac{|ax+by+d|}{\sqrt{a^2+b^2}}$$

$$d_2 = \frac{|2(0)-3(0)+6|}{\sqrt{2^2+3^2}}$$

It is given that, $d_1 = d_2$

$$\frac{8 \times 3}{\sqrt{13b^2}} = \frac{6}{\sqrt{13}}$$

$$b = 4$$

$$\text{So, } a = -2b/3$$

$$= -8/3$$

\therefore The value of a is $-8/3$ and b is 4 .

5. Find the equation to the straight line which cuts off equal positive intercepts on the axes and their product is 25.

Solution:

Given:

$$a = b \text{ and } ab = 25$$

Let us find the equation of the line which cutoff intercepts on the axes.

$$\therefore a^2 = 25$$

$$a = 5 \text{ [considering only positive value of intercepts]}$$

By using the formula,

$$\begin{aligned} \text{The equation of the line with intercepts } a \text{ and } b \text{ is } x/a + y/b &= 1 \\ x/5 + y/5 &= 1 \end{aligned}$$

By taking LCM

$$x + y = 5$$

\therefore The equation of line is $x + y = 5$



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