EXERCISE 13.1

Evaluate the following:

(i) i^{457}

(ii) i^{528}

(iii) $1/ i^{58}$

(iv) $i^{37} + 1/i^{67}$

(v) $[i^{41} + 1/i^{257}]$

(vi) $(i^{77} + i^{70} + i^{87} + i^{414})^3$

(vii) $i^{30} + i^{40} + i^{60}$

(viii) $i^{49} + i^{68} + i^{89} + i^{110}$

Solution:

(i) i^{457}

Let us simplify we get,

$i^{457} = i^{(456 + 1)}$

$= i^{4(114)} \times i$

$= (1)^{114} \times i$

$= i$ [since $i^4 = 1$]

(ii) i^{528}

Let us simplify we get,

$i^{528} = i^{4(132)}$

$= (1)^{132}$

$= 1$ [since $i^4 = 1$]

(iii) $1/ i^{58}$

Let us simplify we get,

$1/ i^{58} = 1/ i^{56+2}$

$= 1/ i^{56} \times i^2$

$= 1/ (i^4)^{14} \times i^2$

$= 1/ i^2$ [since, $i^4 = 1$]

$= 1/-1$ [since, $i^2 = -1$]

$= -1$

(iv) $i^{37} + 1/i^{67}$

Let us simplify we get,

$i^{37} + 1/i^{67} = i^{36+1} + 1/ i^{64+3}$

$= i + 1/i^3$ [since, $i^4 = 1$]

$= i + i/i^4$

$= i + i$

$= 2i$

(v) $[i^{41} + 1/i^{257}]$

Let us simplify we get,

$[i^{41} + 1/i^{257}] = [i^{40+1} + 1/ i^{256+1}]$

$= [i + 1/i]^9$ [since, $1/i = -1$]

$= [i - i]$

$= 0$

(vi) $(i^{77} + i^{70} + i^{87} + i^{414})^3$

Let us simplify we get,

$(i^{77} + i^{70} + i^{87} + i^{414})^3 = (i^{(76 + 1)} + i^{(68 + 2)} + i^{(84 + 3)} + i^{(412 + 2)})^3$

$= (i + i^2 + i^3 + i^2)^3$ [since $i^3 = – i$, $i^2 = – 1$]

$= (i + (– 1) + (– i) + (– 1))^3$

$= (– 2)^3$

$= – 8$

(vii) $i^{30} + i^{40} + i^{60}$

Let us simplify we get,

$i^{30} + i^{40} + i^{60} = i^{(28 + 2) +} i^{40} + i^{60}$

$= (i^4)^7 i^2 + (i^4)^{10} + (i^4)^{15}$

$= i^2 + 1^{10} + 1^{15}$

$= – 1 + 1 + 1$

$= 1$

(viii) $i^{49} + i^{68} + i^{89} + i^{110}$

Let us simplify we get,

$i^{49} + i^{68} + i^{89} + i^{110} = i^{(48 + 1)} + i^{68} + i^{(88 + 1)} + i^{(116 + 2)}$

$= (i^4)^{12}\times i + (i^4)^{17} + (i^4)^{22}\times i + (i^4)^{29}\times i^2$

$= i + 1 + i – 1$

$= 2i$

2. Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number?

Solution:

Given:

$1 + i^{10} + i^{20} + i^{30} = 1 + i^{(8 + 2) +} i^{20} + i^{(28 + 2)}$

$= 1 + (i^4)^2 \times i^2 + (i^4)^5 + (i^4)^7 \times i^2$

$= 1 – 1 + 1 – 1$ [since, $i^4 = 1$, $i^2 = – 1$]

$= 0$

Hence , $1 + i^{10} + i^{20} + i^{30}$ is a real number.

3. Find the values of the following expressions:
(i) $i^{49} + i^{68} + i^{89} + i^{110}$
(ii) $i^{30} + i^{80} + i^{120}$
(iii) $i + i^2 + i^3 + i^4$
(iv) $i^5 + i^{10} + i^{15}$
(v) $[i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}]$
(vi) $1 + i^2 + i^4 + i^6 + i^8 + ... + i^{20}$
(vii) $(1 + i)^6 + (1 – i)^3$
Solution:
(i) $i^{49} + i^{68} + i^{89} + i^{110}$
Let us simplify we get,
$i^{49} + i^{68} + i^{89} + i^{110} = i^{(48 + 1)} + i^{68} + i^{(88 + 1)} + i^{(108 + 2)}$
$= (i^4)^{12} \times i + (i^4)^{17} + (i^4)^{22} \times i + (i^4)^{27} \times i^2$
$= i + 1 + i – 1$ [since $i^4 = 1$, $i^2 = – 1$]
$= 2i$
$\therefore i^{49} + i^{68} + i^{89} + i^{110} = 2i$

(ii) $i^{30} + i^{80} + i^{120}$
Let us simplify we get,
$i^{30} + i^{80} + i^{120} = i^{(28 + 2)} + i^{80} + i^{120}$
$= (i^4)^7 \times i^2 + (i^4)^{20} + (i^4)^{30}$
$= – 1 + 1 + 1$ [since $i^4 = 1$, $i^2 = – 1$]
$= 1$
$\therefore i^{30} + i^{80} + i^{120} = 1$

(iii) $i + i^2 + i^3 + i^4$
Let us simplify we get,
$i + i^2 + i^3 + i^4 = i + i^2 + i^2\times i + i^4$
$= i – 1 + (– 1) \times i + 1$ [since $i^4 = 1$, $i^2 = – 1$]
$= i – 1 – i + 1$
$= 0$
$\therefore i + i^2 + i^3 + i^4 = 0$

(iv) $i^5 + i^{10} + i^{15}$
Let us simplify we get,
$i^5 + i^{10} + i^{15} = i^{(4 + 1)} + i^{(8 + 2)} + i^{(12 + 3)}$
$= (i^4)^1\times i + (i^4)^2\times i^2 + (i^4)^3\times i^3$

$= (i^4)^1 \times i + (i^4)^2 \times i^2 + (i^4)^3 \times i^2 \times i$

$= 1 \times i + 1 \times (-1) + 1 \times (-1) \times i$

$= i - 1 - i$

$= -1$

$\therefore i^5 + i^{10} + i^{15} = -1$

(v) $[i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}]$

Let us simplify we get,

$[i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}]$

$= [i^{10} (i^{582} + i^{580} + i^{578} + i^{576} + i^{574}) / (i^{582} + i^{580} + i^{578} + i^{576} + i^{574})]$

$= i^{10}$

$= i^8 i^2$

$= (i^4)^2 i^2$

$= (1)^2 (-1)$ [since $i^4 = 1$, $i^2 = -1$]

$= -1$

$\therefore [i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}] = -1$

(vi) $1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}$

Let us simplify we get,

$1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20} = 1 + (-1) + 1 + (-1) + 1 + \ldots + 1$

$= 1$

$\therefore 1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20} = 1$

(vii) $(1 + i)^6 + (1 - i)^3$

Let us simplify we get,

$(1 + i)^6 + (1 - i)^3 = \{(1 + i)^2\}^3 + (1 - i)^2 (1 - i)$

$= \{1 + i^2 + 2i\}^3 + (1 + i^2 - 2i)(1 - i)$

$= \{1 - 1 + 2i\}^3 + (1 - 1 - 2i)(1 - i)$

$= (2i)^3 + (-2i)(1 - i)$

$= 8i^3 + (-2i) + 2i^2$

$= -8i - 2i - 2$ [since $i^3 = -i$, $i^2 = -1$]

$= -10i - 2$

$= -2(1 + 5i)$

$= -2 - 10i$

$\therefore (1 + i)^6 + (1 - i)^3 = -2 - 10i$