

Exercise 13(C)

Given a triangle ABC in which A = (4, -4), B = (0, 5) and C = (5, 10). A point P lies on BC such that BP: PC = 3: 2. Find the length of line segment AP.

Solution:

Given, BP: PC = 3: 2

Then by section formula, the co-ordinates of point P are given as:

$$\left(\frac{3 \times 5 + 2 \times 0}{3 + 2}, \frac{3 \times 10 + 2 \times 5}{3 + 2} \right)$$

$$= (15/5, 40/5)$$

$$= (3, 8)$$

Now, by using distance formula, we get

$$AP = \sqrt{(3 - 4)^2 + (8 + 4)^2} = \sqrt{1 + 144} = \sqrt{145} = 12.04$$

1. A (20, 0) and B (10, -20) are two fixed points. Find the co-ordinates of a point P in AB such that: 3PB = AB. Also, find the co-ordinates of some other point Q in AB such that AB = 6AQ.

Solution:

Given, 3PB = AB

So,

$$AB/PB = 3/1$$

$$(AB - PB)/PB = (3 - 1)/1$$

$$AP/PB = 2/1$$

By section formula, we get the coordinates of P to be

$$P(x, y) = P\left(\frac{2 \times 10 + 1 \times 20}{2 + 1}, \frac{2 \times (-20) + 1 \times 0}{2 + 1} \right)$$

$$= P(40/3, -40/3)$$

Also given that, AB = 6AQ

$$AQ/AB = 1/6$$

$$AQ/(AB - AQ) = 1/(6 - 1)$$

$$AQ/QB = 1/5$$

Now, again by using section formula we get

The coordinates of Q as

$$Q(x, y) = Q\left(\frac{1 \times 10 + 5 \times 20}{1 + 5}, \frac{1 \times (-20) + 5 \times 0}{1 + 5} \right)$$

$$= Q(110/6, -20/6)$$

$$= Q(55/3, -10/3)$$

2. A (-8, 0), B (0, 16) and C (0, 0) are the vertices of a triangle ABC. Point P lies on AB and Q lies on AC such that AP: PB = 3: 5 and AQ: QC = 3: 5. Show that: PQ = 3/8 BC.

Solution:

Given that, point P lies on AB such that AP: PB = 3: 5.

So, the co-ordinates of point P are given as

$$\left(\frac{3 \times 0 + 5 \times (-8)}{3 + 5}, \frac{3 \times 16 + 5 \times 0}{3 + 5} \right)$$

$$= (-40/8, 48/8)$$

$$= (-5, 6)$$

Also given that, point Q lies on AB such that AQ: QC = 3: 5.

So, the co-ordinates of point Q are given as

$$\left(\frac{3 \times 0 + 5 \times (-8)}{3 + 5}, \frac{3 \times 0 + 5 \times 0}{3 + 5} \right)$$

$$= (-40/8, 0/8)$$

$$= (-5, 0)$$

Now, by distance formula we get

$$PQ = \sqrt{(-5 + 5)^2 + (0 - 6)^2} = \sqrt{0 + 36} = 6$$

$$BC = \sqrt{(0 - 0)^2 + (0 - 16)^2} = \sqrt{0 + (16)^2} = 16$$

Thus,

$$3/8 \times BC = 3/8 \times 16 = 6 = PQ$$

Hence proved.

3. Find the co-ordinates of points of trisection of the line segment joining the point (6, -9) and the origin.

Solution:

Let's assume P and Q to be the points of trisection of the line segment joining A (6, -9) and B (0, 0).

So, P divides AB in the ratio 1: 2.

Hence, the co-ordinates of point P are given as

$$\left(\frac{1 \times 0 + 2 \times 6}{1 + 2}, \frac{1 \times 0 + 2 \times (-9)}{1 + 2} \right)$$

$$= (12/3, -18/3)$$

$$= (4, -6)$$

And, Q divides AB in the ratio 2: 1.

Hence, the co-ordinates of point Q are

$$\left(\frac{2 \times 0 + 1 \times 6}{2 + 1}, \frac{2 \times 0 + 1 \times (-9)}{2 + 1} \right)$$

$$= (6/3, -9/3) = (2, -3)$$

Therefore, the required coordinates of trisection of PQ are (4, -6) and (2, -3).

4. A line segment joining A (-1, 5/3) and B (a, 5) is divided in the ratio 1: 3 at P, point where the line segment AB intersects the y-axis.

(i) Calculate the value of 'a'.

(ii) Calculate the co-ordinates of 'P'.

Solution:

As, the line segment AB intersects the y-axis at point P, let the co-ordinates of point P be taken as (0, y).

And, P divides AB in the ratio 1: 3.

So,

$$(0, y) = \left(\frac{1 \times a + 3 \times (-1)}{1 + 3}, \frac{1 \times 5 + 3 \times \frac{5}{3}}{1 + 3} \right)$$

$$(0, y) = (a-3/4, 10/4)$$

$$0 = a-3/4 \quad \text{and} \quad y = 10/4$$

$$a - 3 = 0 \quad \text{and} \quad y = 5/2$$

$$a = 3$$

Therefore, the value of a is 3 and the co-ordinates of point P are $(0, 5/2)$.

5. In what ratio is the line joining A (0, 3) and B (4, -1) divided by the x-axis? Write the co-ordinates of the point where AB intersects the x-axis.

Solution:

Let assume that the line segment AB intersects the x-axis by point P (x, 0) in the ratio k: 1.

$$\therefore (x, 0) = \left(\frac{k \times 4 + 1 \times 0}{k + 1}, \frac{k \times (-1) + 1 \times 3}{k + 1} \right)$$

$$(x, 0) = \left(\frac{4k}{k + 1}, \frac{-k + 3}{k + 1} \right)$$

$$0 = (-k + 3)/(k + 1)$$

$$k = 3$$

Therefore, the required ratio in which P divides AB is 3: 1.

Also,

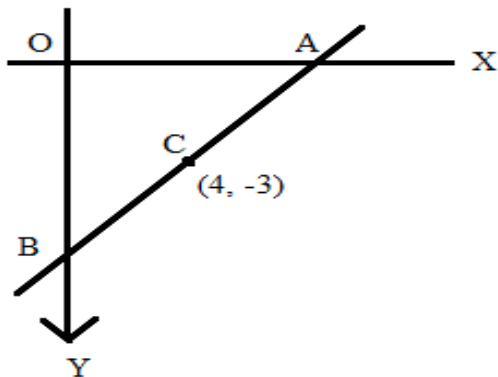
$$x = 4k/(k + 1)$$

$$x = (4 \times 3)/(3 + 1)$$

$$x = 12/3 = 3$$

Hence, the co-ordinates of point P are (3, 0).

6. The mid-point of the segment AB, as shown in diagram, is C (4, -3). Write down the co-ordinates of A and B.



Solution:

Chapter 13: Section and Mid-Point Formula

As, point A lies on x-axis, we can assume the co-ordinates of point A to be (x, 0).

As, point B lies on y-axis, we can assume the co-ordinates of point B to be (0, y).

And given, the mid-point of AB is C (4, -3).

$$\therefore (4, -3) = \left(\frac{x+0}{2}, \frac{0+y}{2} \right)$$

$$(4, -3) = (x/2, y/2)$$

$$4 = x/2 \quad \text{and} \quad -3 = y/2$$

$$x = 8 \quad \text{and} \quad y = -6$$

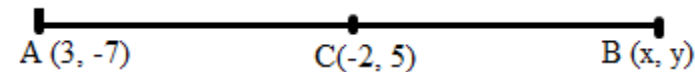
Therefore, the co-ordinates of point A are (8, 0) and the co-ordinates of point B are (0, -6).

7. AB is a diameter of a circle with centre C = (-2, 5). If A = (3, -7), find

(i) the length of radius AC

(ii) the coordinates of B.

Solution:



$$\begin{aligned} \text{(i) Radius AC} &= \sqrt{[(3 + 2)^2 + (-7 - 5)^2]} \\ &= \sqrt{[5^2 + (-12)^2]} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} = 13 \text{ units} \end{aligned}$$

(ii) Let the coordinates of B be (x, y).

Now, by mid-point formula, we get

$$-2 = (3 + x)/2 \quad \text{and} \quad 5 = (-7 + y)/2$$

$$-4 = 3 + x \quad \text{and} \quad 10 = -7 + y$$

$$x = -7 \quad \text{and} \quad y = 17$$

Hence, the coordinates of B are (-7, 17).

8. Find the co-ordinates of the centroid of a triangle ABC whose vertices are:

A (-1, 3), B (1, -1) and C (5, 1)

Solution:

By the centroid of a triangle formula, we get

The co-ordinates of the centroid of triangle ABC as

$$\left(\frac{-1 + 1 + 5}{3}, \frac{3 - 1 + 1}{3} \right)$$

$$= (5/3, 1)$$

9. The mid-point of the line-segment joining (4a, 2b - 3) and (-4, 3b) is (2, -2a). Find the values of a and b.

Solution:

Given that the mid-point of the line-segment joining (4a, 2b - 3) and (-4, 3b) is (2, -2a).

So, we have

$$2 = (4a - 4)/2$$

$$4 = 4a - 4$$

$$8 = 4a$$

$$a = 2$$

Also,

$$-2a = (2b - 3 + 3b)/2$$

$$-2 \times 2 = (5b - 3)/2$$

$$-8 = 5b - 3$$

$$-5 = 5b$$

$$b = -1$$

10. The mid-point of the line segment joining $(2a, 4)$ and $(-2, 2b)$ is $(1, 2a + 1)$. Find the value of a and b .

Solution:

Given,

The mid-point of $(2a, 4)$ and $(-2, 2b)$ is $(1, 2a + 1)$

So, by using mid-point formula, we know

$$(1, 2a + 1) = (2a - 2/2, 4 + 2b/2)$$

$$1 = 2a - 2/2 \quad \text{and} \quad 2a + 1 = 4 + 2b/2$$

$$2 = 2a - 2 \quad \text{and} \quad 4a + 2 = 4 + 2b$$

$$4 = 2a \quad \text{and} \quad 4a = 2 + 2b$$

$$a = 2 \quad \text{and} \quad 4(2) = 2 + 2b \quad [\text{using the value of } a]$$

$$8 = 2 + 2b$$

$$6 = 2b$$

$$b = 3$$

Hence, the value of $a = 2$ and $b = 3$.