NCERT Solutions for Class-XII Maths

Chapter-5.6

NCERT Math Class 12

If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$

1. $x = 2at^2$, $y = at^4$

1. $x = 2at^2, y = at^4$

Here, $x = 2at^2, y = at^4$

Therefore, $\frac{dx}{dt} = 2a(2t)$ and $\frac{dy}{dt} = a(4t^3)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4at^3}{4at} = t^2$$

2. x = a cos θ, y = b cos θ

2. It is given that

x = a cos θ, y = b cos θ

Then, we have

$$\frac{dx}{d\theta} = \frac{d(a\cos\theta)}{d\theta}$$

$= a(-\sin\theta)$

$= -a\sin\theta$ …………………… (1)

$$\frac{dy}{d\theta} = \frac{d(b\cos\theta)}{d\theta}$$

$= b(-\sin\theta)$

$= -b\sin\theta$ …………………… (2)

From equation (1) and (2), we get

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-b\sin\theta}{-a\sin\theta} = \frac{b}{a}$$

Hence, the value of $\frac{dy}{dx}$ is $\frac{b}{a}$

3. x = sin t, y = cos 2t

3. $x = \sin t, y = \cos\ 2t$

Here, $x = \sin t, y = \cos 2t$

Therefore, $\frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = -\sin 2t.2$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{\text{cot}t} = -\frac{2(2\sin t\cos t)}{\cos t} = -4\sin t$$

4. x = 4t, y = $\frac{4}{t}$

4. It is given that

x = 4t, y = $\frac{4}{t}$

Then, we have

$$\frac{dx}{dt} = \frac{d(4t)}{dt}$$

$= 4$ …………………… (1)

$$\frac{dy}{dt} = \frac{d\left(\frac{4}{t}\right)}{dt} = 4\frac{-1}{t^2} = \frac{-4}{t^2} \quad \text{……………………… (2)}$$

Therefore, from equation (1) and (2), we get

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-4}{t^2}}{4} = \frac{-1}{t^2} = -4\sin t$$

Hence, the value of $\frac{dy}{dx}$ is $\frac{-1}{t^2}$

5. x = cos θ – cos 2θ, y = sin θ – sin 2θ

5. $x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

Here, $x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

Therefore, $\frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$ and $\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{\cos\theta-2\cos2\theta}{-\sin\theta+2\sin2\theta}$$

6. x = a (θ – sin θ), y = a (1 + cos θ)

6. It is given that

x = a (θ – sin θ), y = a (1 + cos θ)

Then, we have

$$\frac{dx}{d\theta}=a\left[\frac{d(\theta)}{d\theta}-\frac{d(\sin\theta)}{d\theta}\right]$$

= a(1-cosθ) …………………… (1)

$$\frac{dy}{d\theta}=a\left[\frac{d(1)}{d\theta}-\frac{d(\cos\theta)}{d\theta}\right]$$

= a[0 + (-sinθ)]

= -asinΘ …………………… (2)

From equation (1) and (2), we get

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-a\sin\theta}{a(1-\cos\theta)}$$

$$=\frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$=\frac{-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}=-\cot\frac{\theta}{2}$$

Hence, the value of $\frac{dy}{dx}$ **is** $-\cot\frac{\theta}{2}$

7. $x=\frac{\sin^3 t}{\sqrt{\cos 2t}},y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$

7. Here, $x=\frac{\sin^3 t}{\sqrt{\cos 2t}},y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$

Therefore, $\frac{dx}{dt}=\frac{\sin^3 t\frac{d}{dt}\sqrt{\cos 2t}-\sqrt{\cos 2t}\frac{d}{dt}\sin^3 t}{(\sqrt{\cos 2t})^2}$

$$= \frac{\sin^3 t \cdot \frac{1}{2\sqrt{\cos 2t}}(-\sin 2t)\cdot 2 - \sqrt{\cos 2t}\cdot 3\sin^2 t\cos t}{\cos 2t}$$

$$= \frac{-\sin^3 t \cdot \sin 2t - 3\cos 2t \cdot \sin^2 t\cos t}{\cos 2t\sqrt{\cos 2t}}$$

and $\frac{dy}{dt} = \frac{\cos^3 t \frac{d}{dt}\sqrt{\cos 2t} - \sqrt{\cos 2t}\frac{d}{dt}\cos^3 t}{(\sqrt{\cos 2t})^2}$

$$= \frac{\cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}}(-\sin 2t)\cdot 2 - \sqrt{\cos 2t}\cdot 3\cos^2 t(-\sin t)}{\cos 2t}$$

$$= \frac{-\cos^3 t \cdot \sin 2t + 3\cos 2t \cdot \cos^2 t\sin t}{\cos 2t\sqrt{\cos 2t}}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\cos^3 t \cdot \sin 2t + 3\cos 2t \cdot \cos^2 t\sin t}{-\sin^3 t \cdot \sin 2t - 3\cos 2t \cdot \sin^2 t\cos t}$$

$$= \frac{-\cos^3 t \cdot (2\sin t\cos t) + 3\cos 2t \cdot \cos^2 t\sin t}{-\sin^3 t \cdot (2\sin t\cos t) - 3\cos 2t \cdot \sin^2 t\cos t} = \frac{\cos^2 t\sin t\left(-2\cos^2 t + 3\cos 2t\right)}{\sin^2 t\cos t\left(-2\sin^2 t - 3\cos 2t\right)}$$

$$= \frac{\cos t\left[-2\cos^2 t + 3\left(2\cos^2 t - 1\right)\right]}{\sin t\left[-2\sin^2 t - 3\left(1 - 2\sin^2 t\right)\right]} = \frac{\cos t\left[-2\cos^2 t + 6\cos^2 t - 3\right]}{\sin t\left[-2\sin^2 t - 3 + 6\sin^2 t\right]}$$

$$= \frac{\cos t\left[4\cos^2 t - 3\right]}{\sin t\left[-3 + 4\sin^2 t\right]} = -\frac{4\cos^3 t - 3\cos t}{3\sin t - 4\sin^3 t} = -\frac{\cos 3t}{\sin 3t} = -\cot 3t$$

8. $x = a\left(\cos t + \log\tan\frac{t}{2}\right) y = a\sin t$

8. It is given that

$x = a\left(\cos t + \log\tan\frac{t}{2}\right) y = a\sin t$

Then, we have

$$\frac{dx}{dt} = a\left[\frac{d(\cos t)}{dt} + \frac{d\left(\log\tan\frac{t}{2}\right)}{dt}\right]$$

$$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\frac{d\left(\tan\frac{t}{2}\right)}{dt}\right]$$

$$= a\left[-\sin t + \cot\frac{t}{2}.\sec^2\frac{t}{2}\frac{d\left(\frac{t}{2}\right)}{dt}\right]$$

$$= a\left[-\sin t + \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}} \times \frac{1}{\cos^2\frac{t}{2}} \times \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right]$$

$$= a\left[-\sin t + \frac{1}{\sin t}\right]$$

$$= a\left[\frac{\sin^2 t + 1}{\sin t}\right]$$

$$= a\frac{\cos^2 t}{\sin t} \text{ .... (1)}$$

$$\frac{dy}{dt} = a\frac{d(\sin t)}{dt}$$

$= a\cos t$....................... (2)

From equation (1) and (2), we get

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{\left(a\frac{\cos^2 t}{\sin t}\right)}$$

$$= \frac{\sin t}{\cos t}$$

$= \tan t$

Hence, the value of $\frac{dy}{dx}$ is tant

9. $x = a\sec\theta,\ y = b\tan\theta$

9. It is given that

$x = a \sec\theta,\ y = b\tan\theta$

Then, we have

$$\frac{dx}{d\theta} = a\frac{d(\sec\theta)}{d\theta}$$

$= a\sec\theta\tan\theta$…………………… (1)

$$\frac{dy}{d\theta} = b\frac{d(\tan\theta)}{d\theta}$$

$= b\sec^2\theta$…………………… (2)

From equation (1) and (2), we get,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta}$$

$$= \frac{b}{a}\sec\theta\cot\theta$$

$$= \frac{b\cos\theta}{a\cos\theta\sin\theta}$$

$$= \frac{b}{a}\times\frac{1}{\sin\theta}$$

$$= \frac{b}{a}\text{cosec}\theta$$

Hence, the value of $\frac{dy}{dx}$ is $\frac{b}{a}\text{cosec}\theta$

10. $x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta)$

10. It is given that

$x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta)$

Then, we have

$$\frac{dx}{d\theta} = a\left[\frac{d(\cos\theta)}{d\theta} + \frac{d(\theta\sin\theta)}{d\theta}\right]$$

$$= a\left[-\sin\theta + \frac{\theta d(\sin\theta)}{d\theta} + \sin\theta\frac{d(\theta)}{d\theta}\right]$$

$= a[-\sin\theta + \theta\cos\theta + \sin\theta]$

$= a\theta\cos\theta$…………………… (1)

$$\frac{dy}{d\theta} = a\left[\frac{d(\sin\theta)}{d\theta} - \frac{d(\theta\cos\theta)}{d\theta}\right]$$

$$= a\left[\cos\theta - \left\{\frac{\theta d(\cos\theta)}{d\theta} + \cos\theta\frac{d(\theta)}{d\theta}\right\}\right]$$

= a[cosθ + θsinθ - cosθ]

= aθsinθ…………………… (2)

From (1) and (2) we get,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta\sin\theta}{a\theta\cos\theta}$$

= tanθ

Hence, the value of $\frac{dy}{dx}$ **is** tanθ

11. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

11. ere, $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$

Therefore,

$$\frac{dx}{dt} = \frac{1}{2\sqrt{a^{\sin^{-1}t}}} \cdot \frac{d}{dx} a^{\sin^{-1}t} = \frac{1}{2\sqrt{a^{\sin^{-1}t}}} \cdot a^{\sin^{-1}t} \cdot \log a \frac{1}{\sqrt{1-t^2}}$$

$$= \frac{1}{2x} \cdot x^2 \cdot \log a \frac{1}{\sqrt{1-t^2}} = \frac{x\log a}{\sqrt{1-t^2}}$$ and $$\frac{dy}{dt} = \frac{1}{2\sqrt{a^{\cos^{-1}t}}} \cdot \frac{d}{dx} a^{\cos^{-1}t}$$

$$= \frac{1}{2\sqrt{a^{\cos^{-1}t}}} \cdot a^{\cos^{-1}t} \cdot \log a \frac{-1}{\sqrt{1-t^2}}$$

$$= \frac{1}{2y} \cdot y^2 \cdot \log a \frac{1}{\sqrt{1-t^2}} = -\frac{y\log a}{\sqrt{1-t^2}}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\frac{y\log a}{\sqrt{1-t^2}}}{\frac{x\log a}{\sqrt{1-t^2}}} = -\frac{y}{x}$$