

### EXERCISE 4.13

1. If  $\cos^{-1}(x/2) + \cos^{-1}(y/3) = \alpha$ , then prove that  $9x^2 - 12xy \cos \alpha + 4y^2 = 36 \sin^2 \alpha$

**Solution:**

Given  $\cos^{-1}(x/2) + \cos^{-1}(y/3) = \alpha$

We know that,

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$$

Now by substituting, we get

$$\Rightarrow \cos^{-1}\left[\frac{x}{2} \times \frac{y}{3} - \sqrt{1-\left(\frac{x}{2}\right)^2} \sqrt{1-\left(\frac{y}{3}\right)^2}\right] = \alpha$$

$$\Rightarrow \left[\frac{xy}{6} - \frac{\sqrt{4-x^2}}{2} \times \frac{\sqrt{9-y^2}}{3}\right] = \cos \alpha$$

$$\Rightarrow xy - \sqrt{4-x^2} \times \sqrt{9-y^2} = 6 \cos \alpha$$

$$\Rightarrow xy - 6 \cos \alpha = \sqrt{4-x^2} \sqrt{9-y^2}$$

On squaring both the sides we get

$$\Rightarrow (xy - 6 \cos \alpha)^2 = (4-x^2)(9-y^2)$$

$$\Rightarrow x^2y^2 + 36\cos^2\alpha - 12xy \cos \alpha = 36 - 9x^2 - 4y^2 + x^2y^2$$

$$\Rightarrow 9x^2 + 4y^2 - 36 + 36\cos^2\alpha - 12xy \cos \alpha = 0$$

$$\Rightarrow 9x^2 + 4y^2 - 12xy \cos \alpha - 36(1 - \cos^2 \alpha) = 0$$

$$\Rightarrow 9x^2 + 4y^2 - 12xy \cos \alpha - 36\sin^2 \alpha = 0$$

$$\Rightarrow 9x^2 + 4y^2 - 12xy \cos \alpha = 36\sin^2 \alpha$$

Hence, proved.

**2. Solve the equation:  $\cos^{-1}(a/x) - \cos^{-1}(b/x) = \cos^{-1}(1/b) - \cos^{-1}(1/a)$**

**Solution:**

Given  $\cos^{-1}(a/x) - \cos^{-1}(b/x) = \cos^{-1}(1/b) - \cos^{-1}(1/a)$

$$\Rightarrow \cos^{-1} \frac{a}{x} + \cos^{-1} \frac{1}{a} = \cos^{-1} \frac{1}{b} + \cos^{-1} \frac{b}{x}$$

We know that,

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{1-x^2}\sqrt{1-y^2}]$$

By substituting this formula we get,

$$\Rightarrow \cos^{-1} \left[ \frac{1}{x} - \sqrt{1 - \left(\frac{a}{x}\right)^2} \sqrt{1 - \left(\frac{1}{a}\right)^2} \right] = \cos^{-1} \left[ \frac{1}{x} - \sqrt{1 - \left(\frac{b}{x}\right)^2} \sqrt{1 - \left(\frac{1}{b}\right)^2} \right]$$

$$\Rightarrow \frac{1}{x} - \sqrt{1 - \left(\frac{a}{x}\right)^2} \sqrt{1 - \left(\frac{1}{a}\right)^2} = \frac{1}{x} - \sqrt{1 - \left(\frac{b}{x}\right)^2} \sqrt{1 - \left(\frac{1}{b}\right)^2}$$

$$\Rightarrow \sqrt{1 - \left(\frac{a}{x}\right)^2} \sqrt{1 - \left(\frac{1}{a}\right)^2} = \sqrt{1 - \left(\frac{b}{x}\right)^2} \sqrt{1 - \left(\frac{1}{b}\right)^2}$$

Squaring on both the sides, we get

$$\Rightarrow \left(1 - \left(\frac{a}{x}\right)^2\right) \left(1 - \left(\frac{1}{a}\right)^2\right) = \left(1 - \left(\frac{b}{x}\right)^2\right) \left(1 - \left(\frac{1}{b}\right)^2\right)$$

$$\Rightarrow 1 - \left(\frac{a}{x}\right)^2 - \left(\frac{1}{a}\right)^2 + \left(\frac{1}{x}\right)^2 = 1 - \left(\frac{b}{x}\right)^2 - \left(\frac{1}{b}\right)^2 + \left(\frac{1}{x}\right)^2$$

$$\Rightarrow \left(\frac{b}{x}\right)^2 - \left(\frac{a}{x}\right)^2 = \left(\frac{1}{a}\right)^2 - \left(\frac{1}{b}\right)^2$$

On simplifying, we get

$$\Rightarrow (b^2 - a^2) a^2 b^2 = x^2 (b^2 - a^2)$$

$$\Rightarrow x^2 = a^2 b^2$$

$$\Rightarrow x = a b$$



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