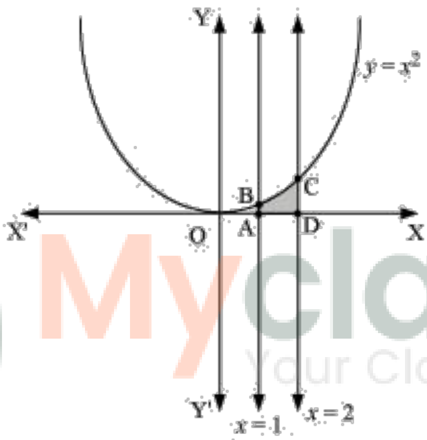
NCERT Solutions for Class-XII Math

Chapter-8
Exercise- Miscellaneous
NCERT Maths Class 12

1. Find the area under the given curves and given lines:
(i) $y = x^2$, $x = 1$, $x = 2$ and x-axis
(ii) $y = x^4$, $x = 1$, $x = 5$ and x –axis

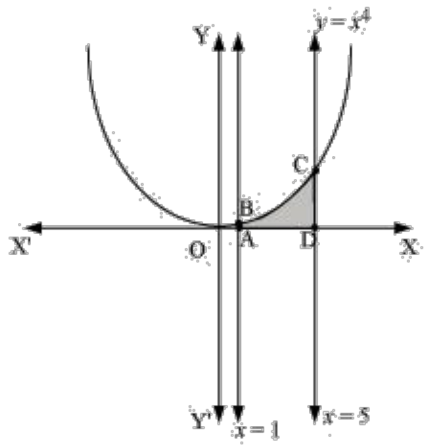
1. (i) The required area is represented by the shaded area ADCBA as



$$\text{Area ADCBA} = \int_1^2 y\,dx$$

$$= \int_1^2 x^2 dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \text{units}$$

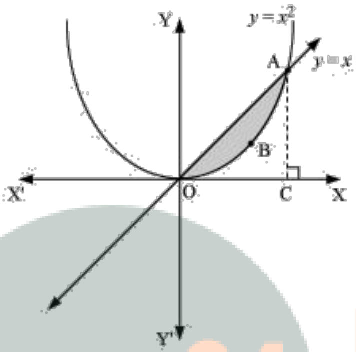
(ii) The required area is represented by the shaded area ADCBA as



Area ADCBA $= \int_1^5 x^4 dx$

$$= \left[\frac{x^5}{5}\right]_1^5 = \frac{(5)^5}{5} - \frac{1}{5} = 625 - \frac{1}{5} \qquad = 624.8 \text{ units}$$

2. Find the area between the curves y = x and y = x^2.



2.

We can see from the figure that the area of the region bounded by the curve y = x and y = $x^2$and points of intersection are A (1, 1).

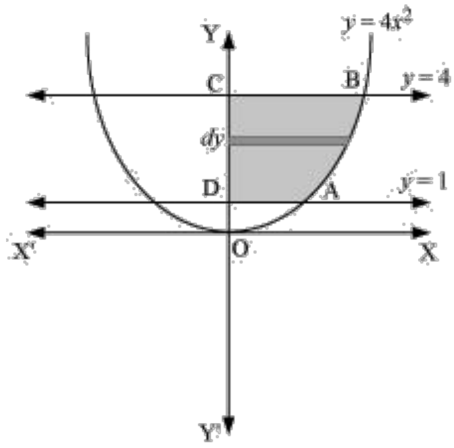
Thus,

Area of OBAO = Area (ΔOCA) – Area (OCABO)

$$= \int_0^1 x dx - \int_0^1 x^2 dx = \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}\text{units.}$$

3. Find the area of the region lying in the first quadrant and bounded by y = $4x^2$, x = 0, y = 1 and y = 4

3. The area in the first quadrant bounded by y = $4x^2$, x = 0, y = 1, and y = 4 is represented by the shaded area ABCDA as



$$\therefore \text{ Area ADCD} = \int_1^4 x\,dx$$

$$= \int_1^4 \frac{\sqrt{y}}{2} dx$$

$$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$$= \frac{1}{3}\left[(4)^{\frac{3}{2}} - 1\right]$$

$$= \frac{1}{3}[8-1]$$

$$= \frac{7}{3} \text{units}$$

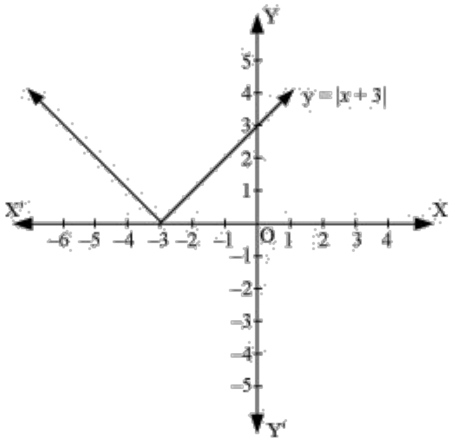
4. Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3| dx$

4. It is given that y = |x + 3|

The value of x and y are given in the following table:

X	-6	-5	-4	-3	-2	-1	0
Y	3	2	1	0	1	2	3

By plotting these points, we get the graph of y = |x+3| as below:



We know that, (x + 3) ≤ 0 for -6 ≤ x ≤ -3 and (x + 3) ≥ 0 for -3 ≤ x ≤ 0.

Thus, $\int_{-6}^{0}|x+3|dx = -\int_{-6}^{-3}(x+3)dx + \int_{-3}^{0}(x+3)dx$

$= -\left[\frac{x^2}{2}+3x\right]_{-6}^{-3} + \left[\frac{x^2}{2}+3x\right]_{-3}^{0}$

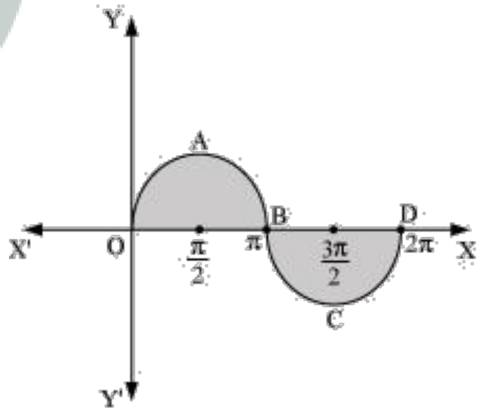
$= -\left[\left(\frac{(-3)^2}{2}+3(-3)\right)-\left(\frac{(-6)^2}{2}+3(-6)\right)\right]+\left[0-\left(\frac{(-3)^2}{2}+3(-3)\right)\right]$

$= -\left[-\frac{9}{2}\right]-\left[-\frac{9}{2}\right]$

= 9

5. Find the area bounded by the curve y = sin x between x = 0 and x = 2π

5. The graph of y = sin x can be drawn as



∴ Required area = Area OABO + Area BCDB

$= \int_0^{\pi}\sin x dx + \left|\int_{\pi}^{2\pi}\sin x dx\right|$

$= [-\cos x]_0^{\pi} + \left|[-\cos x]_{\pi}^{2\pi}\right|$

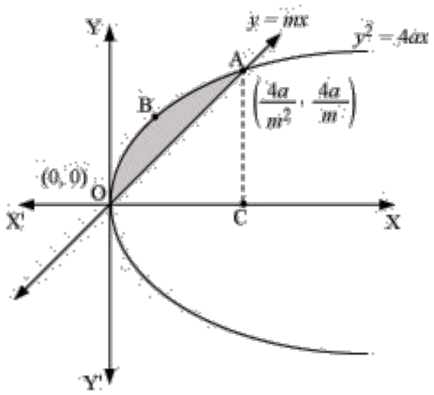
= [−cosπ + cos0] + |−cos2π + cosπ|

= 1 + 1 + |(−1−1)|

= 2 + |−2|

= 2 + 2 = 4 units

6. Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$.



6. We can see from the figure that the area of the region bounded by the curve $y^2 = 4ax$ and the line $y = mx$ is shown by shaded region that is Area OABO.

The points of intersection of both the curves are (0,0) and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.

Now draw AC perpendicular to x – axis.

Thus,

Area of OABO = Area OCABO – Area (ΔOCA)

$$= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax}dx - \int_0^{\frac{4a}{m^2}} mxdx$$

$$= 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{4a}{m^2}} - m\left[\frac{x^2}{2}\right]_0^{\frac{4a}{m^2}}$$

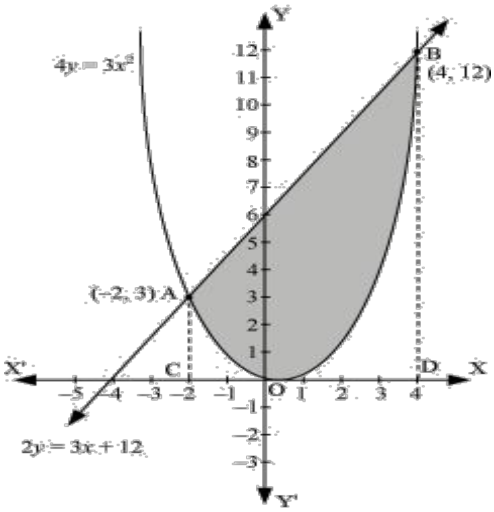
$$= \frac{4}{3}\sqrt{a}\left(\frac{4a}{m^2}\right)^{\frac{3}{2}} - \frac{m}{2}\left(\frac{4a}{m^2}\right)^2$$

$$= \frac{32a^2}{3m^3} - \frac{m}{2}\left(\frac{16a^2}{m^4}\right)$$

$$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3} = \frac{8a^2}{3m^3} units$$

7. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

7. The area enclosed between the parabola, $4y = 3x^2$, and the line, $2y = 3x + 12$, is represented by the shaded area OBAO as



The points of intersection of the given curves are A (–2, 3) and (4, 12).

We draw AC and BD perpendicular to x-axis.

$\therefore$ Area OBAO = Area CDBA – (Area ODBO + Area OACO)

$$= \int_{-2}^{4} \frac{1}{2}(3x+12)\,dx - \int_{-2}^{4} \frac{3x^2}{4}\,dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2}+12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$$

$$= \frac{1}{2}[24+48-6+24] - \frac{1}{4}[64+8]$$

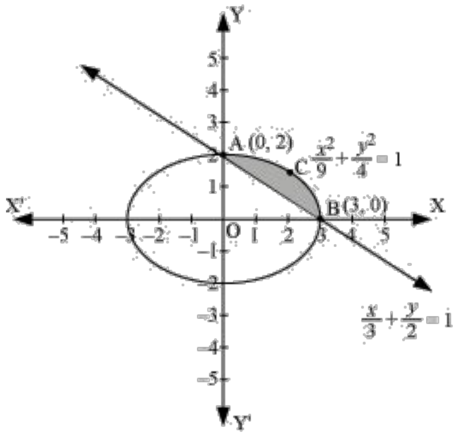
$$= \frac{1}{2}[90] - \frac{1}{4}[72]$$

= 45 – 18

= 27 units

8. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$

8.



The area of the smaller region bounded by the ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$

Area of BCAB = Area OBCAO – Area OBAO

$$=\int_0^3 2\sqrt{1-\frac{x^2}{9}}\,dx-\int_0^3 2\left(1-\frac{x}{3}\right)dx$$

$$=\frac{2}{3}\left[\int_0^3 \sqrt{9-x^2}dx\right]-\frac{2}{3}\int_0^3(3-x)dx$$

$$=\frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3-\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$$

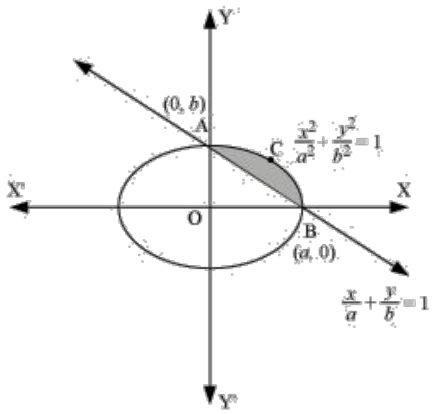
$$=\frac{2}{3}\left[\frac{9}{2}\left(\frac{\pi}{2}\right)\right]-\frac{2}{3}\left[9-\frac{9}{2}\right]$$

$$=\frac{2}{3}\left[\frac{9\pi}{4}-\frac{9}{2}\right]$$

$$=\frac{2}{3}\times\frac{9}{4}[\pi-2]=\frac{3}{2}(\pi-2)units$$

9. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.

9. The area of the smaller region bounded by the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and the line, $\frac{x}{a}+\frac{y}{b}=1$, is represented by the shaded region BCAB as



$\therefore$ Area BCAB = Area (OBCAO) – Area (OBAO)

$$= \int_0^a b\sqrt{1-\frac{x^2}{a^2}}dx - \int_0^a b\left(1-\frac{x}{a}\right)dx$$

$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}dx - \frac{b}{a}\int_0^a (a-x)dx$$

$$= \frac{b}{a}\left[\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}_0^a - \left\{ax-\frac{x^2}{2}\right\}_0^a\right]$$

$$= \frac{b}{a}\left[\left\{\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right\} - \left\{a^2-\frac{a^2}{2}\right\}\right]$$

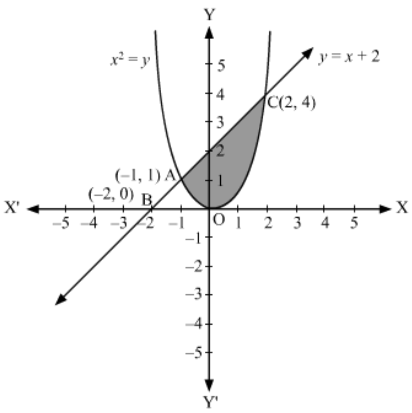
$$= \frac{b}{a}\left[\frac{a^2\pi}{4}-\frac{a^2}{2}\right]$$

$$= \frac{ba^2}{2a}\left[\frac{\pi}{2}-1\right]$$

$$= \frac{ab}{2}\left[\frac{\pi}{2}-1\right]$$

$$= \frac{ab}{4}[\pi-2]$$

10. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and x axis.



10.

We can see from the figure that the area of the region bounded by the parabola $x^2 = y$, the line y = x + 2 and the x-axis is shown by shaded region that is Area OACO.

The points of intersection of both the curves are A (-1, 1) and C (2, 4).

Thus,

Area of OACO

$= \int_{-1}^{2}(x+2)dx - \int_{-1}^{2} x^2 dx$

$= \left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{3}[x^3]_{-1}^{2}$

$= \left[\left\{\frac{(2)^2}{2} + 2(2)\right\} - \left\{\frac{(-1)^2}{2} + 2(-1)\right\}\right] - \frac{1}{3}[(2)^3 - (-1)^3]$

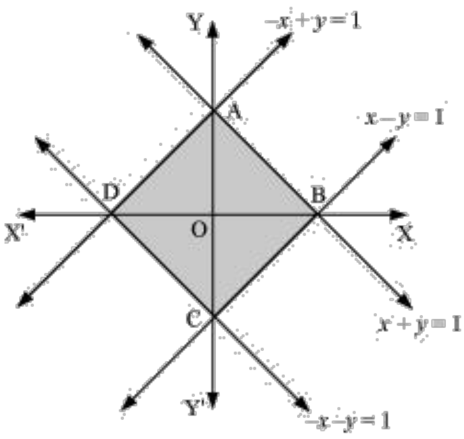
$= \left[2 + 4 - \left\{\frac{1}{2} - 2\right\}\right] - \frac{1}{3}[8 - 1]$

$= \left[6 + \frac{3}{2} - 3\right] = \frac{9}{2} units$

11. Using the method of integration find the area bounded by the curve $|x| + |y| = 1$.

[Hint: the required region is bounded by lines x + y = 1, x – y = 1, – x + y = 1 and – x – y = 11]

11. The area bounded by the curve, $|x| + |y| = 1$, is represented by the shaded region ADCB as



The curve intersects the axes at points A (0, 1), B (1, 0), C (0, –1), and D (–1, 0).

It can be observed that the given curve is symmetrical about x-axis and y-axis.

$\therefore$ Area ADCB = 4 × Area OBAO

$= 4\int_0^1 (1-x)dx$

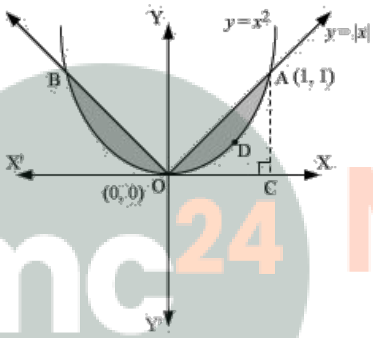
$= 4\left(x - \frac{x^2}{2}\right)_0^1$

$= 4\left[1 - \frac{1}{2}\right]$

$= 4\left(\frac{1}{2}\right)$

= 2 units

12. Find the area bounded by curves $\{(x, y): y \geq x^2$ and $y = |x|\}$.



12. We can observed that the required area is symmetrical about y –axis.

Required area = 2[Area OCAO – Area OCADO]

$= 2\left[\int_0^1 xdx - \int_0^1 x^2dx\right]$

$= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right]$

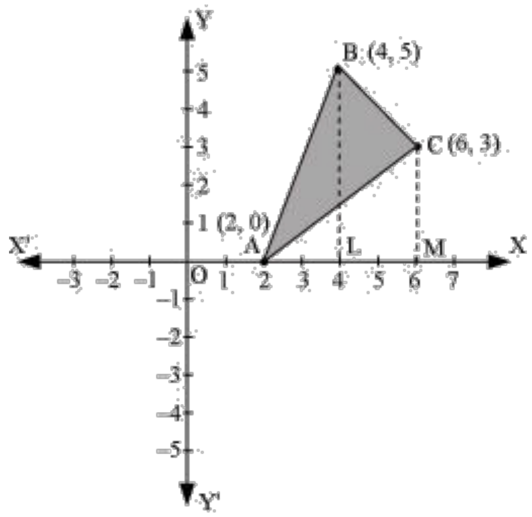
$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$

$= 2\left(\frac{1}{6}\right) = \frac{1}{3}$

13. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are

A (2, 0), B (4, 5) and C (6, 3).

13. The vertices of ΔABC are A (2, 0), B (4, 5), and C (6, 3).



Equation of line segment AB is

$$y-0=\frac{5-0}{4-2}(x-2)$$

2y = 5x – 10

$$y=\frac{5}{2}(x-2) \qquad \dots (1)$$

Equation of line segment BC is

$$y-5=\frac{3-5}{6-4}(x-4)$$

2y – 10 = –2x + 8

2y = –2x + 18

y = –x + 9 … (2)

Equation of line segment CA is

$$y-3=\frac{0-3}{2-6}(x-6)$$

–4y + 12 = –3x + 18

4y = 3x – 6

$$y=\frac{3}{4}(x-2) \qquad \dots (3)$$

Area (ΔABC) = Area (ABLA) + Area (BLMCB) – Area (ACMA)

$$=\int_2^4 \frac{5}{2}(x-2)dx+\int_4^6(-x+9)dx-\int_2^6\frac{3}{4}(x-2)dx$$

$$=\frac{5}{2}\left[\frac{x^2}{2}-2x\right]_2^4+\left[\frac{-x^2}{2}+9x\right]_4^6-\frac{3}{4}\left[\frac{x^2}{2}-2x\right]_2^6$$

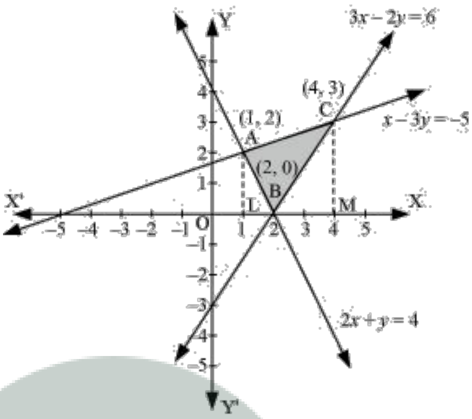
$$=\frac{5}{2}[8-8-2+4]+[-18+54+8-36]-\frac{3}{4}[18-12-2+4]$$

$= 5 + 8 - \frac{4}{4}(8)$

$= 13 - 6$

$= 7$ units

14. Using the method of integration find the area of the region bounded by lines:
$2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$



14.

It is given lines if equations are $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.
Thus, the area of the region bounded by the lines is the area of ΔABC.
And let us draw AL and CM perpendicular to x – axis.
Then,
Area (ΔABC) = Area ALMCA – Area ALB – Area CMB

$= \int_1^4 \left(\frac{x+5}{3}\right) dx - \int_1^2 (4-2x)dx - \int_2^4 \left(\frac{3x-6}{2}\right) dx$

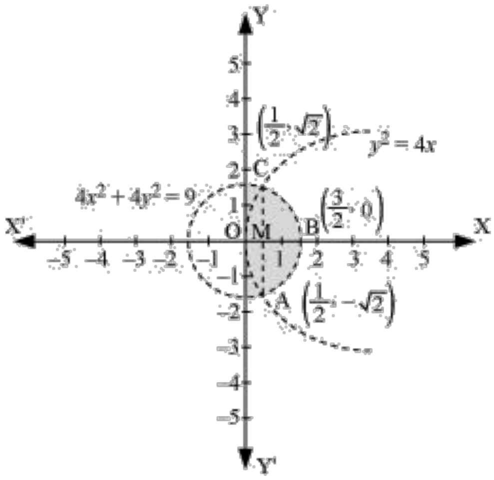
$= \frac{1}{3}\left[\frac{x^2}{2} + 5x\right]_1^4 - [-4x - x^2]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2} - 6x\right]_2^4$

$= \frac{1}{3}\left[8 + 20 - \frac{1}{2} - 5\right] - [8 - 4 - 4 + 1] - \frac{1}{2}[24 - 24 - 6 + 12]$

$= \left(\frac{1}{3} \times \frac{45}{2}\right) - (1) - \frac{1}{2}(6) = \frac{15}{2} - 1 - 3 = \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2}$

15. Find the area of the region $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

15. Find the area of the region $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$, is represented as



The points of intersection of both the curves are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$.

The required area is given by OABCO.
It can be observed that area OABCO is symmetrical about x-axis
$\therefore$ Area OABCO = 2 × Area OBC
Area OBCO = Area OMC + Area MBC

$$= \int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{9 - 4x^2}\,dx$$

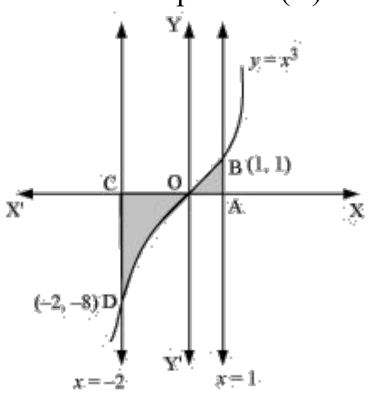
$$= \int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{(3)^2 - (2x)^2}\,dx$$

16. Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

(a) -9 (b) $-\frac{15}{4}$

(c) $\frac{15}{4}$ (d)

16. The correct option is (B).



Here, the required area $= \int_{-2}^{1} y\,dx$

$= \int_{-2}^{1} x^3 dx$

$= \left[\frac{x^4}{4}\right]_{-2}^{1}$

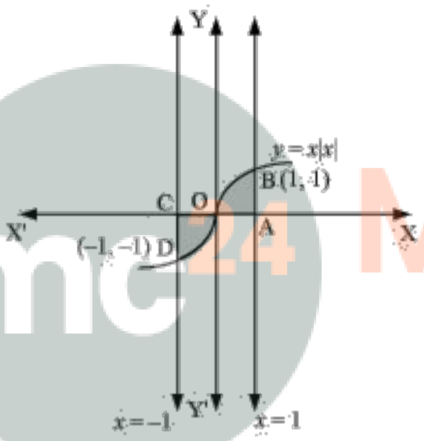
$= \left[\frac{1}{4} - \frac{(-2)^4}{4}\right]$

$= \left(\frac{1}{4} - 4\right) = -\frac{15}{4} units.$

17. The area bounded by the curve $y = x|x|$, x-axis and the ordinates x = –1 and x = 1 is given by
[Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

(a) 0 (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{4}{3}$



17.

Required area $= \int_{-1}^{1} y dx$

$= \int_{-1}^{1} x|x| dx$

$= \int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$

$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$

$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$

$= \frac{2}{3}$ units

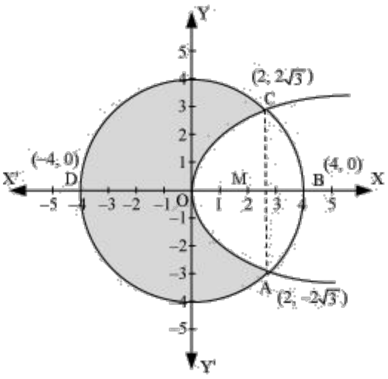
Thus, the correct answer is C.

18. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

(a) $\frac{4}{3}\left(4\pi-\sqrt{3}\right)$ (b) $\frac{4}{3}\left(4\pi+\sqrt{3}\right)$

(c) $\frac{4}{3}\left(8\pi-\sqrt{3}\right)$ (d) $\frac{4}{3}\left(4\pi+\sqrt{3}\right)$

18. The correct option is (C).



Here, the area bounded by the circle and parabola = 2 × [Area OADO + Area ADBA]

$= 2\left[\int_0^2 \sqrt{16xdx} + \int_2^4 \sqrt{16-x^2}dx\right]$

$= 2\left[\sqrt{6}\left\{\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right\}_0^2\right] + 2\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$

$= 2\sqrt{6} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^2 + 2\left[8.\frac{\pi}{2} - \sqrt{16-4} - 8\sin^{-1}\left(\frac{1}{2}\right)\right]$

$= \frac{4\sqrt{6}}{3}\left(2\sqrt{2}\right) + 2\left[4\pi - \sqrt{12} - 8\frac{\pi}{6}\right]$

$= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3}\pi$

$= \frac{4}{3}\left[4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi\right]$

$= \frac{4}{3}\left[\sqrt{3} + 4\pi\right]$

Area of circle = π(r2)

= $\pi(4)^2$

=16π units.

Thus, required area = $16\pi - \frac{4}{3}\left[4\pi + \sqrt{3}\right]$

$= \frac{4}{3}\left[4 \times 3\pi - 4\pi - \sqrt{3}\right]$

$= \frac{4}{3}\left[8\pi - \sqrt{3}\right] units$

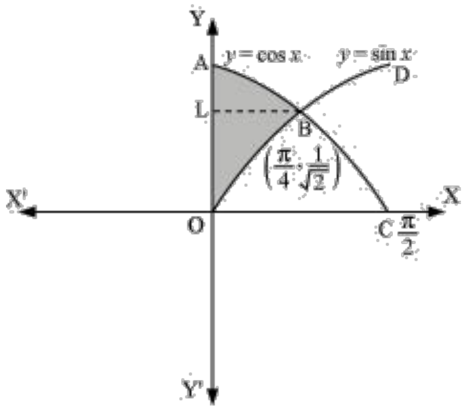
19. The area bounded by the y-axis, y = cos x and y = sin x when $0 \le x \le \frac{\pi}{2}$.

(a) $2\left(\sqrt{2}-1\right)$ (b) $\sqrt{2}-1$

(c) $\sqrt{2}+1$ (d) $\sqrt{2}$

19. The correct option is (B).

The given equations are y = cos x … (1)
And, y = sin x … (2)



Required area = Area (ABLA) + area (OBLO)

$$= \int_{\frac{1}{\sqrt{2}}}^{1} x\,dy + \int_{0}^{\frac{1}{\sqrt{2}}} x\,dy$$

$$= \int_{\frac{1}{\sqrt{2}}}^{1} \cos^{-1} y\,dy + \int_{0}^{\frac{1}{\sqrt{2}}} \sin^{-1} x\,dy$$

Integrating by parts, we obtain

$$= \left[y\cos^{-1} y - \sqrt{1-y^2}\right]_{\frac{1}{\sqrt{2}}}^{1} + \left[x\sin^{-1} x + \sqrt{1-x^2}\right]_{0}^{\frac{1}{\sqrt{2}}}$$

$$= \left[\cos^{-1}(1) - \frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}}\right] + \left[\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}-1}\right]$$

$$= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

$$= \frac{2}{\sqrt{2}} - 1$$

$$= \sqrt{2} - 1 \text{ units}$$

Thus, the correct answer is B.