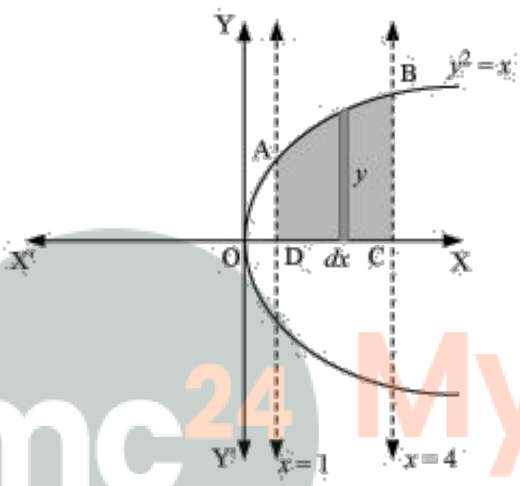
NCERT Solutions for Class-XII Maths

Chapter-8.1

NCERT Maths Class 12

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis.



1.

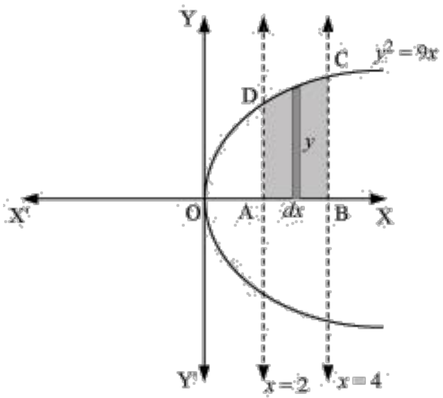
The area of the region bounded by the curve, $y^2 = x$, the lines, $x = 1$ and $x = 4$, and the x-axis is the area ABCD.

Area of ABCD $= \int_1^4 y\,dx$

$$= \int_1^4 \sqrt{x}\,dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 = \frac{2}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right] = \frac{2}{3}[8-1] = \frac{14}{3}\text{units}$$

2. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

2.



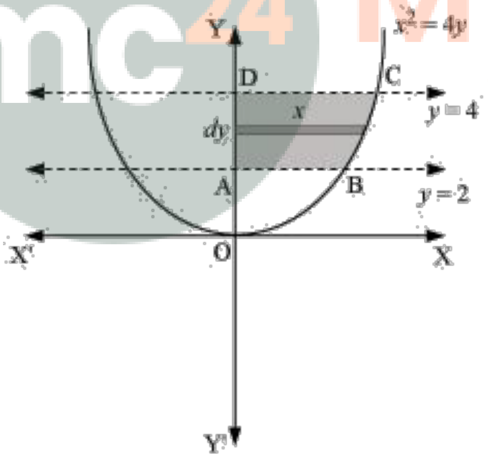
We can see from the figure that the area of the region bounded by the curve $y^2 = 9x$, $x = 2$, $x = 4$ is shown by shaded region that is Area ABCD.

Area of ABCD = $\int_2^4 y dy = 3\int_2^4 \sqrt{x} dx$

$$= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4 = 2\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right] = \frac{2}{3}[8 - 2\sqrt{2}] = \left(16 - 4\sqrt{2}\right)\text{units}$$

3. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

3.



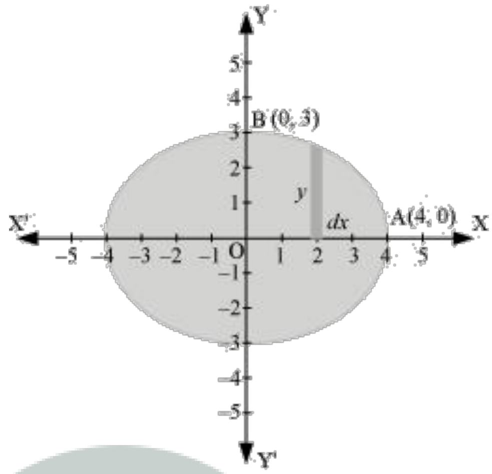
The area of the region bounded by the curve, $x^2 = 4y$, $y = 2$, and $y = 4$, and the y-axis is the area ABCD.

Area of ABCD $= \int_2^4 x dy$

$$= \int_2^4 2\sqrt{y} dy = 2\int_2^4 \sqrt{y} dy = 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$$

$$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right] = \frac{4}{3}\left[8 - 2\sqrt{2}\right] = \left(\frac{32 - 8\sqrt{2}}{3}\right)\text{units}$$

4. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.



4.

It is given that equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

We can see that the ellipse is symmetrical about x – axis and y –axis.

∴ Area bounded by ellipse = 4 × Area of OAB

Area of ABCD = $\int_0^1 y dx = \int_0^1 3\sqrt{1 - \frac{x^2}{16}} dx$

$= \frac{3}{4}\int_0^1 \sqrt{16 - x^2} dx$

$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$= \frac{3}{4}\left[2\sqrt{16 - 16} + 8\sin^{-1}(1) - 0 - 8\sin^{-1}(0)\right]$

$= \frac{3}{4}\left[\frac{8\pi}{2}\right]$

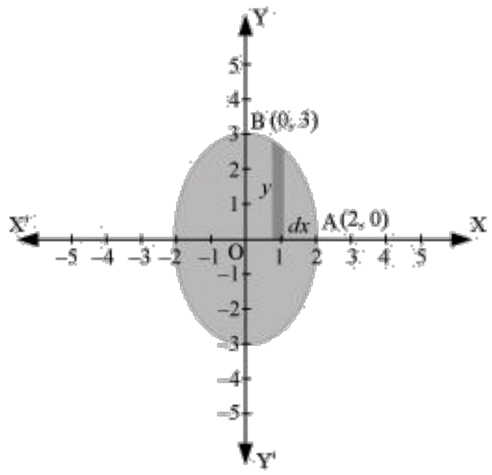
$= \frac{3}{4}[4\pi]$

$= 3\pi$

Therefore, the required area bounded by the ellipse = 4 × 3π = 12π units.

5. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

5. The given equation of the ellipse can be represented as



$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\Rightarrow y=3\sqrt{1-\frac{x^2}{4}} \qquad \ldots (1)$$

It can be observed that the ellipse is symmetrical about x-axis and y-axis.

$\therefore$ Area bounded by ellipse = 4 × Area OAB

$\therefore$ Area of ABCD $=\int_0^2 y dx$

$$=\int_0^2 3\sqrt{1-\frac{x^2}{4}}dx \qquad \text{[Using (1)]}$$

$$=\frac{3}{2}\int_0^2 3\sqrt{4-x^2}dx$$

$$=\frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin\frac{-x}{2}\right]_0^2$$

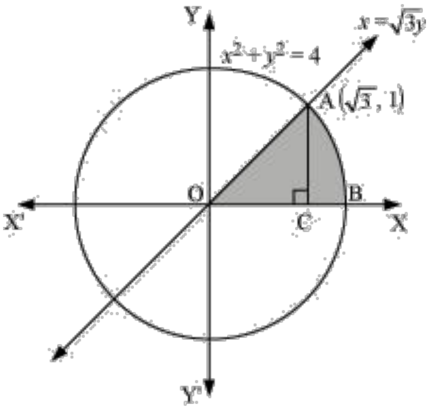
$$=\frac{3}{2}\left[\frac{2\pi}{2}\right]$$

$$=\frac{3\pi}{2}$$

Therefore, area bounded by the ellipse $=4\times\frac{3\pi}{2}=6\pi$ units

6. Find the area of the region in the first quadrant enclosed by x-axis, line $x=\sqrt{3}y$ and the circle
$x^2 + y^2 = 4$.

6.



The equations are $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

From the figure we can see that the x-axis is the area OAB and it si shown by shaded region.

Now, the point of intersection of the line and the circle in the first quadrant is$(\sqrt{3}, 1)$.

Area OAB = Area ΔOCA + Area ACB

Area of ΔOCA $= \frac{1}{2} \times \mathrm{OC} \times \mathrm{AC} = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$

Also,

Area of ABC $= \int_{\sqrt{3}}^{2} y\,dx = \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\,dx$

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$

$= \left[\pi - \frac{\sqrt{3}\pi}{2}\sqrt{4 - 3} - 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \left[\pi - \frac{\sqrt{3}}{2} - 2\left(\frac{\pi}{3}\right)\right]$

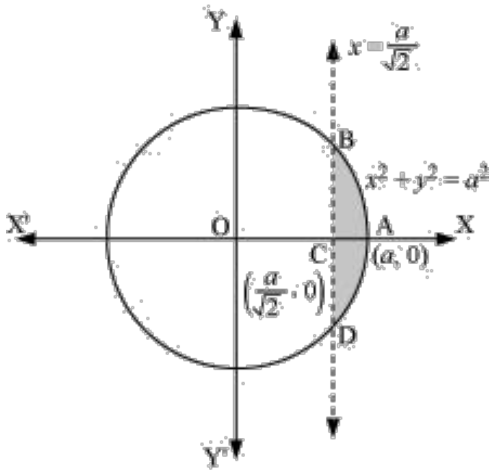
$= \left[\pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right]$

$= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right]$

Therefore, required area is $= \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{2} = \frac{\pi}{3}$ square units.

7. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

7. The area of the smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is the area ABCDA.



It can be observed that the area ABCD is symmetrical about x-axis.

$\therefore$ Area ABCD = 2 × Area ABC

Area of ABC $=\int_{\frac{a}{\sqrt{2}}}^{a} y\,dx$

$$=\int_{\frac{a}{\sqrt{2}}}^{a}\sqrt{a^2-x^2}\,dx=\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$$

$$=\left[\frac{a^2}{2}\left(\frac{\pi}{2}\right)-\frac{a^2}{2\sqrt{2}}\sqrt{a^2-\frac{a^2}{2}}-\frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]=\frac{a^2\pi}{4}-\frac{a}{2\sqrt{2}}\cdot\frac{a}{\sqrt{2}}-\frac{a^2}{\pi}\left(\frac{\pi}{4}\right)$$

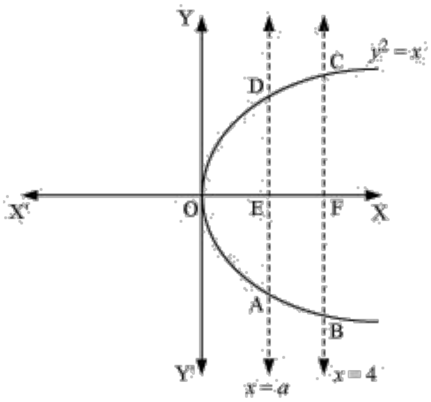
$$=\frac{a^2\pi}{4}-\frac{a^2}{4}-\frac{a^2\pi}{8}=\frac{a^2}{4}\left[\pi-1-\frac{\pi}{2}\right]=\frac{a^2}{4}\left[\frac{\pi}{2}-1\right]$$

Area ABCD $=2\left[\frac{a^2}{4}\left(\frac{\pi}{2}-1\right)\right]=\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$

Therefore, the area of smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x=\frac{a}{\sqrt{2}}$ is $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$ units.

8. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a.

8.



It is given that the area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$.

Thus, Area OAD = Area ABCD

Now, we can observed that the given area is symmetrical about x-axis.

Area OED = Area EFCD

Now, Area of OED = $\int_0^a y dx = \int_0^a \sqrt{x} dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a = \frac{2}{3}(a)^{\frac{3}{2}}$……………….(1)

Area of EFCD = $\int_a^4 y dx = \int_a^4 \sqrt{x} dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_a^4 = \frac{2}{3}\left[8 - (a)^{\frac{3}{2}}\right]$……………….(2)

Therefore, from equations (1) and (2), we get,

$\frac{2}{3}(a)^{\frac{3}{2}} = \frac{2}{3}\left[8 - (a)^{\frac{3}{2}}\right]$

$\Rightarrow 2.(a)^{\frac{3}{2}} = 8$

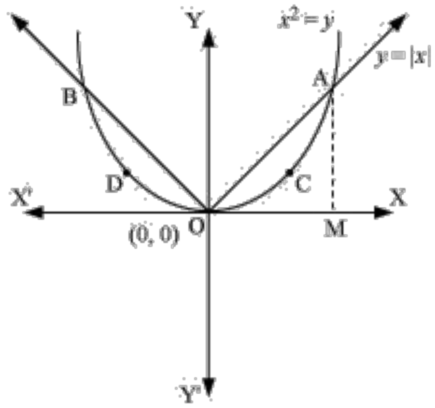
$\Rightarrow (a)^{\frac{3}{2}} = 4$

$\Rightarrow a = (4)^{\frac{3}{2}}$

Hence, the required value of a is $(4)^{\frac{3}{2}}$.

9. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

9. The area bounded by the parabola, $x^2 = y$, and the line, $y = |x|$, can be represented as



The given area is symmetrical about y-axis.

$\therefore$ Area OACO = Area ODBO

The point of intersection of parabola, $x^2 = y$, and line, $y = x$, is A (1, 1).

Area of OACO = Area ΔOAB – Area OBACO

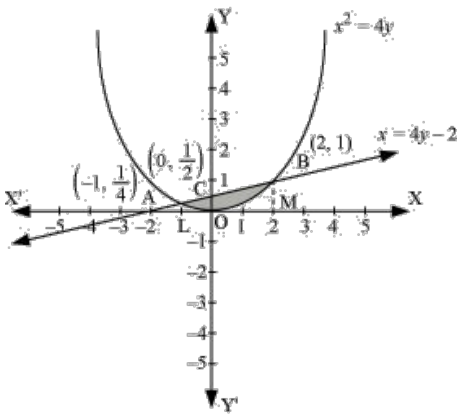
$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times OB \times AB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of OBACO $\int_0^1 y dx = \int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$

$\Rightarrow$ Area of OACO = Area of ΔOAB – Area of OBACO $= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Therefore, required area $= 2\left[\frac{1}{6}\right] = \frac{1}{3}$ units

10. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y – 2$.



10.

It is given that the area of the region bounded by the parabola $x^2 = 4y$ and $x = 4y - 2$.

Let A and B be the points of intersection of the line and parabola.

Coordinates of point A are $\left(-1, \frac{1}{4}\right)$.

Coordinates of point B are (2, 1).

Now, draw AL and BM perpendicular to x axis.

We can see that

Area OBAO = Area OBCO + Area OAC) … (1)

Now, Area OBCA = Area OMBC – Area of OMBO

$$= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx = \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^2 = \frac{1}{4}[2+4] - \frac{1}{4}\left[\frac{8}{3}\right] = \frac{3}{2} - \frac{2}{3} = \frac{5}{6}$$

Similarly, Area OACO = Area OLAC – Area of OLAO

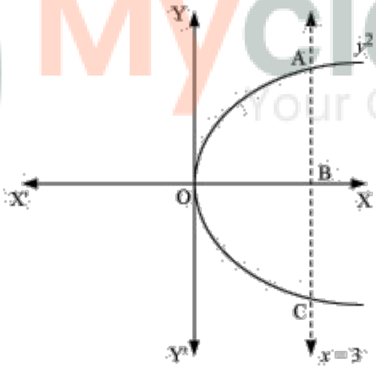
$$= \int_{-1}^0 \frac{x+2}{4} dx - \int_{-1}^0 \frac{x^2}{4} dx = \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^0 - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^0$$

$$= \frac{1}{4}\left[\frac{(-1)^2}{2} + 2(-1)\right] - \frac{1}{4}\left[\frac{(-1)^3}{3}\right] = \frac{1}{4}\left[\frac{1}{2} - 2\right] - \frac{1}{12} = \frac{1}{2} - \frac{1}{8} - \frac{1}{12} = \frac{7}{24}$$

Therefore, the required area is $= \left(\frac{5}{6} + \frac{7}{24}\right) = \frac{9}{8}$ *units.*

11. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$

11. The region bounded by the parabola, $y^2 = 4x$, and the line, $x = 3$, is the area OACO.



The area OACO is symmetrical about x-axis.

∴ Area of OACO = 2 (Area of OAB)

$$\text{Area of OACO} = 2\left[\int_0^3 y\,dx\right] = 2\int_0^3 2\sqrt{x}\,dx = 4\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^3 = \frac{8}{3}\left[(3)^{\frac{3}{2}}\right] = 8\sqrt{3}$$

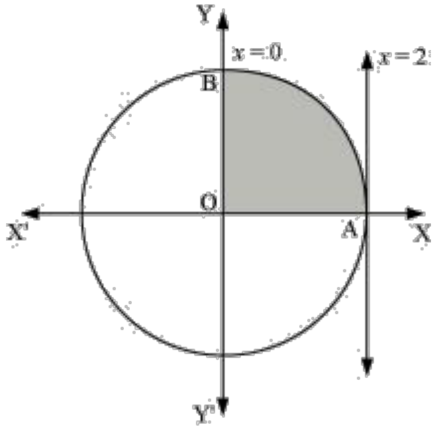
Therefore, the required area is $8\sqrt{3}$ units.

12. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(a) n (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

12. The correct option is (a).



The area bounded by the circle and the lines, x = 0 and x = 2, in the first quadrant is shown by shaded region in above figure.

Area of OAB = $\int_0^2 y dx = \int_0^2 \sqrt{4 - x^2} dx$

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 = 2\left(\frac{\pi}{2}\right) = \pi$

Therefore, required area is = π square units.

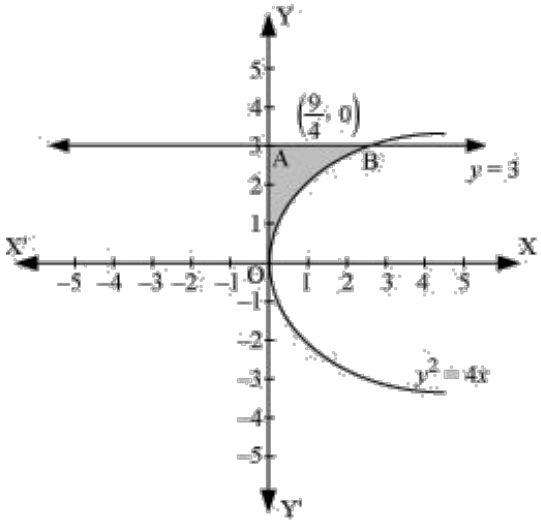
13. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line y = 3 is

(a) 2 (b) $\frac{9}{4}$

(c) $\frac{9}{3}$ (d) $\frac{9}{2}$

13. The correct option is (b).

The area bounded by the curve, $y^2 = 4x$, y-axis, and y = 3 is represented as



$\therefore$ Area of OAB $= \int_0^3 x\,dy$

$$= \int_0^3 \frac{y^2}{4}\,dy$$

$$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$$

$$= \frac{1}{12}(27)$$

$$= \frac{9}{4}\text{ units}$$

Thus, the correct answer is B.