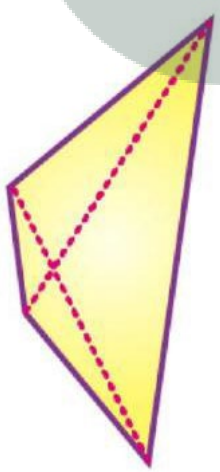
EXERCISE 14B

1. State, 'true' or 'false'
(i) The diagonals of a rectangle bisect each other.
(ii) The diagonals of a quadrilateral bisect each other.
(iii) The diagonals of a parallelogram bisect each other at right angle.
(iv) Each diagonal of a rhombus bisects it.
(v) The quadrilateral, whose four sides are equal, is a square.
(vi) Every rhombus is a parallelogram.
(vii) Every parallelogram is a rhombus.
(viii) Diagonals of a rhombus are equal.
(ix) If two adjacent sides of a parallelogram are equal, it is a rhombus.
(x) If the diagonals of a quadrilateral bisect each other at right angle, the quadrilateral is a square.

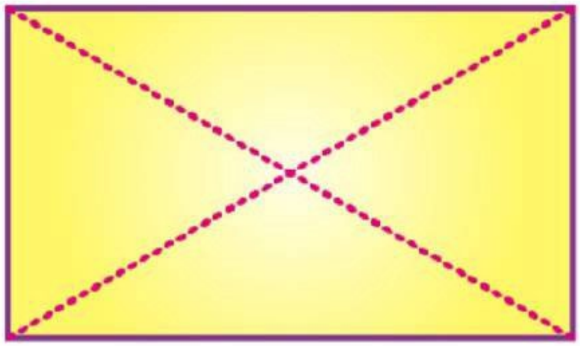
Solution:
(i) True.
This is true, because we know that a rectangle is a parallelogram. So, all the properties of a parallelogram are true for a rectangle. Since the diagonals of a parallelogram bisect each other, the same holds true for a rectangle.

(ii) False
This is not true for any random quadrilateral. Observe the quadrilateral shown below.



Clearly the diagonals of the given quadrilateral do not bisect each other. However, if the quadrilateral was a special quadrilateral like a parallelogram, this would hold true.

(iii) False
Consider a rectangle as shown below.

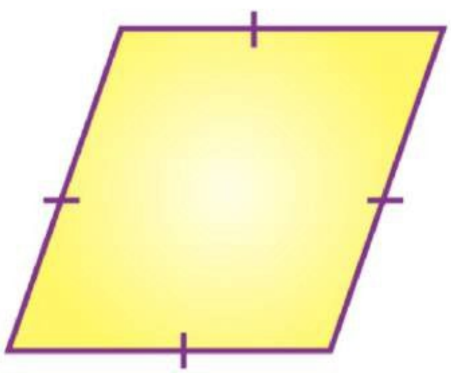


It is a parallelogram. However, the diagonals of a rectangle do not intersect at right angles, even though they bisect each other.

(iv) True
Since a rhombus is a parallelogram, and we know that the diagonals of a parallelogram bisect each other, hence the diagonals of a rhombus too, bisect other.

(v) False
This need not be true, since if the angles of the quadrilateral are not right angles, the quadrilateral would be a rhombus rather than a square.

(vi) True



A parallelogram is a quadrilateral with opposite sides parallel and equal.
Since opposite sides of a rhombus are parallel, and all the sides of the rhombus are equal, a rhombus is a parallelogram.

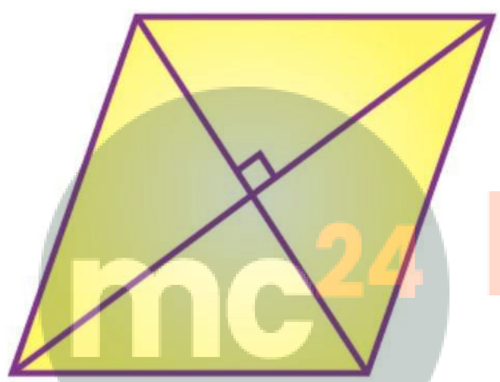
(vii) False
This is false, since a parallelogram in general does not have all its sides equal. Only opposite sides of a parallelogram are equal. However, a rhombus has all its sides equal. So, every parallelogram cannot be

a rhombus, except those parallelograms that have all equal sides.

(viii) False
This is a property of a rhombus. The diagonals of a rhombus need not be equal.

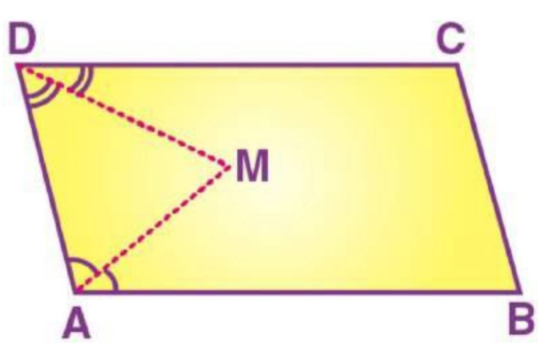
(ix) True
A parallelogram is a quadrilateral with opposite sides parallel and equal.
A rhombus is a quadrilateral with opposite sides parallel, and all sides equal.
If in a parallelogram the adjacent sides are equal, it means all the sides of the parallelogram are equal, thus forming a rhombus.

(x) False



Observe the above figure. The diagonals of the quadrilateral shown above bisect each other at right angles, however the quadrilateral need not be a square, since the angles of the quadrilateral are clearly not right angles.

2. In the figure, given below, AM bisects angle A and DM bisects angle D of parallelogram ABCD. Prove that: ∠AMD = 90°.



Solution:

From the given figure we can conclude that

$\angle A + \angle D = 180^o$ [since consecutive angles are supplementary]

$\angle A/2 + \angle D/2 = 90$

Again, from triangle ADM

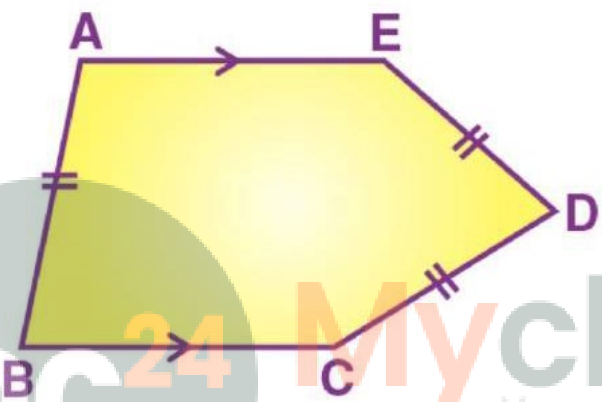
$\angle A/2 + \angle D/2 + \angle M = 180^o$

$90^o + \angle M = 180^o$

$\angle M = 180^o = 90^o$

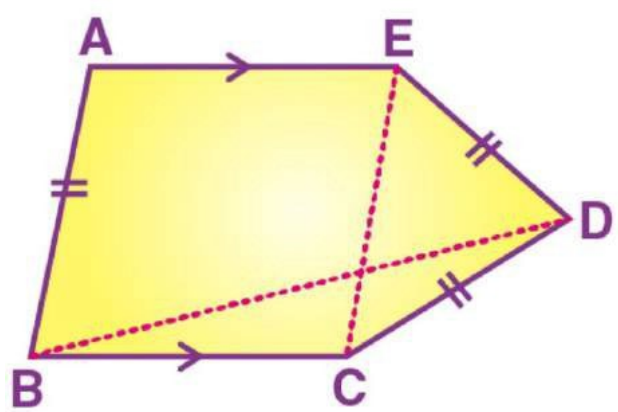
Hence $\angle AMD = 90^o$

3. In the following figure, AE and BC are equal and parallel and the three sides AB, CD and DE are equal to one another. If angle A is 102°. Find angles AEC and BCD.



Solution:

According to the question,



Given that AE = BC

We have to find

$\angle AEC$ and $\angle BCD$

Let us join EC and BD

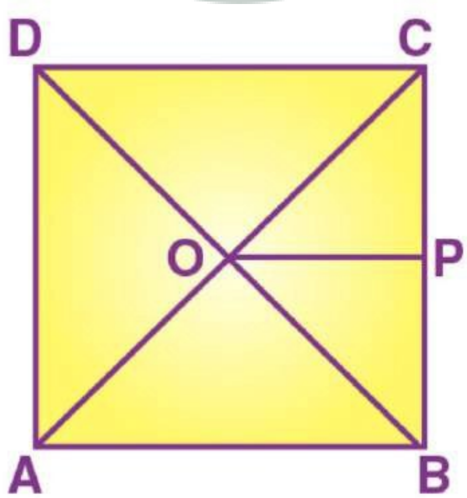
In the quadrilateral AECB

AE = BC and AB = EC

Also, AE parallel to BC
So quadrilateral is a parallelogram.
In parallelogram consecutive angles are supplementary
$\angle A + \angle B = 180^o$
$102^o + \angle B = 180^o$
$\angle B = 78^o$
In parallelogram opposite angles are equal
$\angle A = \angle BEC$ and $\angle B = \angle AEC$
$\angle BEC = 102^o$ and $\angle AEC = 78^o$
Now consider triangle ECD
EC = ED = CD [since AB = EC]
Therefore, triangle ECD is an equilateral triangle.
$\angle ECD = 60^o$
$\angle BCD = \angle BEC + \angle ECD$
$\angle BCD = 102^o + 60^o$
$\angle BCD = 162^o$
Therefore, $\angle AEC = 78^o$ and $\angle BCD = 162^o$

4. In a square ABCD, diagonals meet at O. P is a point on BC such that OB = BP.
Show that:
(i) $\angle POC = 22\ ½^o$
(ii) $\angle BDC = 2\ \angle POC$
(iii) $\angle BOP = 3\ \angle CPO$

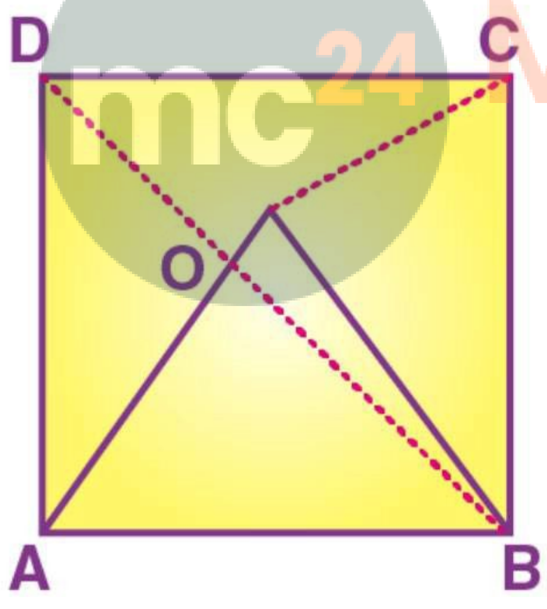
Solution:
Given ABCD is a square and diagonal meet at o. P is a point on BC such that OB = BP



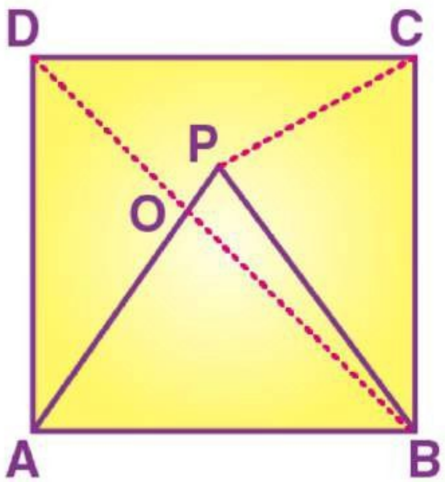
In the triangle BOC and triangle DOC
BD = BD [common side]
BO = CO
OD = OC [since diagonals cuts at O]

△ BOC ≅ △ DOC [By SSS postulate]
Therefore,
∠BOC = 90°
Now ∠POC = 22.5
∠BOP = 67.5
Again, in triangle BDC
∠BDC = 45° [since ∠B = 45° and ∠C = 90°]
Therefore
∠BDC = 2∠POC
∠BOP = 67.5°
∠BOP = 2 ∠POC
Hence the proof.

5. The given figure shows a square ABCD and an equilateral triangle ABP. Calculate:
(i) ∠AOB
(ii) ∠BPC
(iii) ∠PCD
(iv) Reflex ∠APC



Solution:



In the given figure, triangle APB is an equilateral triangle
Therefore, all its angles are 60^o
Again, in the triangle ADB
$\angle ABD = 45^o$
$\angle AOB = 180^o - 60^o - 45^o$
$= 75^o$
Again, in triangle BPC
$\angle BPC = 75^o$ [since BP = CB]
Now,
$\angle C = \angle BCP + \angle PCD$
$\angle PCD = 90^o - 75^o$
$\angle PCD = 15^o$
Therefore
$\angle APC = 60^o + 75^o$
$\angle APC = 135^o$
Reflex $\angle APD = 360^o - 135^o = 225^o$
Therefore
(i) $\angle AOB = 75^o$
(ii) $\angle BPC = 75^o$
(iii) $\angle PCD = 15^o$
(iv) Reflex $\angle APC = 225^o$