EXERCISE 13.4

A cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick. If the outer diameter is 16 cm and its length is 100 cm, find how many cubic centimeters of iron has been used in making the tube?

Solution:

According to the question,
Outer diameter d = 16cm
Then,
Outer radius r – 16/2 = 8cm
Height = length = 100cm
Thickness of iron sheet = 2cm
Volume of cylinder = $\pi r^2 h$, where r= outer radius and $\pi = 3.14$
Thus, Volume of cylinder = $\pi r^2 h$
$= 3.14 \times (8)^2 \times 100$
$= 20{,}096\ cm^3$
Now, inner diameter = outer diameter – 2× thickness of iron sheet
Inner diameter = 16-(2× 2) = 12cm
Inner radius R = 12/2 = 6cm
Thus, Volume of hollow space = $\pi R^2 h$, where R= inner radius and $\pi = 3.14$
$= \pi R^2 h$
$= 3.14 \times (6)^2 \times 100$
$= 11{,}304\ cm^3$

Thus,
Volume of iron used = Volume of cylinder – Volume of hollow space
$= (20{,}096 - 11{,}304)\ cm^3$
$= 8800\ cm^3$

1. A semi-circular sheet of metal of diameter 28cm is bent to form an open conical cup. Find the capacity of the cup.

Solution:

According to the question,
Diameter of semi circular sheet = 28cm
Radius of semi circular sheet (r) = 28/2
= 14cm
Semi Circular sheet is bent to form an open conical cup
Thus, slant height of a conical cup (l)=radius of semicircular sheet(r) = 14cm
We know that,
Circumference of base of a cone = 2πR,
(where, R = radius of a cone and circumference of semi circle = πr)
Thus,
Circumference of base of a cone = Circumference of a Semi circle
$\Rightarrow 2\pi R = \pi r$

$$\Rightarrow R = \frac{\pi r}{2\pi} = \frac{r}{2} = \frac{14}{2} = 7cm$$

To find height,
$R^2 + h^2 = l^2$
Where,
R=radius of a cone
h=height of a cone
l=slant height of a cone
$\Rightarrow (7)^2 + h^2 = (14)^2$
$\Rightarrow 49 + h^2 = 196$
$\Rightarrow h^2 = 196 - 49 = 147$
$\Rightarrow h = \sqrt{147} = 7\sqrt{3}$ cm
Thus, volume of a cone = $1/3\ \pi R^2 h$

$$\Rightarrow \text{Volume of a cone} = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 7\sqrt{3} = \frac{1078\sqrt{3}}{3}\ \text{cm}^3 = \frac{1078}{\sqrt{3}}\ \text{cm}^3$$

$$\Rightarrow \text{Volume of a cone} = \frac{1078}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\ \text{cm}^3 = \frac{1078\sqrt{3}}{3}\ \text{cm}^3$$

$$\Rightarrow \text{Volume of a cone} = \frac{1078\sqrt{3}}{3}\ \text{cm}^3 = 622.38\ \text{cm}^3$$

2. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 m
(i) How many students can sit in the tent if a student, on an average, occupies 5/7 m^2 on the ground?
(ii) Find the volume of the cone.
Solution:

According to the question,
Area of cloth = $165m^2$
Radius of conical tent = 5m
Area covered by 1 student = 5/7 m^2
Curved surface area of cone = πrl
Thus, curved surface area of a conical; tent = πrl

$$\Rightarrow 165 = \frac{22}{7} \times 5 \times l$$

$$\Rightarrow l = \frac{165 \times 7}{22 \times 5} = \frac{21}{2} = 10.5\ \text{m}$$

(i)

$$\text{No. of students} = \frac{\text{Area of circular base of a cone}}{\text{Area covered by 1 student}}$$

$$\Rightarrow \text{No. of student} = \frac{\pi r^2}{\frac{5}{7}} = \frac{\left(\frac{22}{7} \times 5^2\right)}{\frac{5}{7}} = 22 \times 5 = 110$$

$$\text{No. of student occupies} \frac{5}{7}\ \text{m}^2\ \text{of area} = 110$$

(ii) Height of a cone,
$r^2 + h^2 = l^2$
Where,
r=radius of a cone
h=height of a cone
l=slant height of a cone
$\Rightarrow (5)^2 + h^2 = (10.5)^2$
$\Rightarrow 25 + h^2 = 110.25$
$\Rightarrow h^2 = 110.25 - 25 = 85.25$
$\Rightarrow h = \sqrt{85.25} = 9.23$ m
Volume of a come = $(1/3)\ \pi r^2 h$

$$\text{Volume of a cone} = \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 9.23 = \frac{5076.5}{21} = 241.73\ \text{m}^3$$

3. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill to its capacity. Calculate the volume of water pumped into the tank.
Solution:

According to the question,
Internal diameter of hemispherical tank = 14 m
Internal radius of hemispherical tank = 14/2 m = 7m
Tank contains 50kl = $50m^3$ of water (since, 1kl = $1m^3$)
Volume of a hemispherical tank = $2/3\ \pi r^3$

$$\text{Volume of a hemispherical tank} = \frac{2}{3} \times \frac{22}{7} \times (7)^3 = \frac{2156}{3} = 718.66\ \text{m}^3$$

Volume of water pumped into the tank = Volume of hemispherical tank – $50m^3$
= 718.66 – 50
= 668.66 m^3