EXERCISE 15.1

Solve the following linear Inequations in R.

1. Solve: 12x < 50, when
(i) x ∈ R
(ii) x ∈ Z
(iii) x ∈ N
Solution:
Given:
12x < 50
So when we divide by 12, we get
12x/ 12 < 50/12
x < 25/6
(i) x ∈ R
When x is a real number, the solution of the given inequation is (-∞, 25/6).

(ii) x ∈ Z
When, 4 < 25/6 < 5
So when, when x is an integer, the maximum possible value of x is 4.
The solution of the given inequation is {…, –2, –1, 0, 1, 2, 3, 4}.

(iii) x ∈ N
When, 4 < 25/6 < 5
So when, when x is a natural number, the maximum possible value of x is 4. We know that the natural numbers start from 1, the solution of the given inequation is {1, 2, 3, 4}.

2. Solve: -4x > 30, when
(i) x ∈ R
(ii) x ∈ Z
(iii) x ∈ N
Solution:
Given:
-4x > 30
So when we divide by 4, we get
-4x/4 > 30/4
-x > 15/2
x < - 15/2
(i) x ∈ R

When x is a real number, the solution of the given inequation is (-∞, -15/2).

(ii) x ∈ Z
When, -8 < -15/2 < -7
So when, when x is an integer, the maximum possible value of x is -8.
The solution of the given inequation is {…, –11, –10, -9, -8}.

(iii) x ∈ N
As natural numbers start from 1 and can never be negative, when x is a natural number, the solution of the given inequation is ∅.

3. Solve: 4x-2 < 8, when
(i) x ∈ R
(ii) x ∈ Z
(iii) x ∈ N
Solution:
Given:
4x – 2 < 8
4x – 2 + 2 < 8 + 2
4x < 10
So divide by 4 on both sides we get,
4x/4 < 10/4
x < 5/2
(i) x ∈ R
When x is a real number, the solution of the given inequation is (-∞, 5/2).

(ii) x ∈ Z
When, 2 < 5/2 < 3
So when, when x is an integer, the maximum possible value of x is 2.
The solution of the given inequation is {…, –2, –1, 0, 1, 2}.

(iii) x ∈ N
When, 2 < 5/2 < 3
So when, when x is a natural number, the maximum possible value of x is 2. We know that the natural numbers start from 1, the solution of the given inequation is {1, 2}.

4. 3x – 7 > x + 1
Solution:
Given:

$3x - 7 > x + 1$
$3x - 7 + 7 > x + 1 + 7$
$3x > x + 8$
$3x - x > x + 8 - x$
$2x > 8$
Divide both sides by 2, we get
$2x/2 > 8/2$
$x > 4$
∴ The solution of the given inequation is $(4, \infty)$.

5. $x + 5 > 4x - 10$
Solution:
Given: $x + 5 > 4x - 10$
$x + 5 - 5 > 4x - 10 - 5$
$x > 4x - 15$
$4x - 15 < x$
$4x - 15 - x < x - x$
$3x - 15 < 0$
$3x - 15 + 15 < 0 + 15$
$3x < 15$
Divide both sides by 3, we get
$3x/3 < 15/3$
$x < 5$
∴ The solution of the given inequation is $(-\infty, 5)$.

6. $3x + 9 \geq -x + 19$
Solution:
Given: $3x + 9 \geq -x + 19$
$3x + 9 - 9 \geq -x + 19 - 9$
$3x \geq -x + 10$
$3x + x \geq -x + 10 + x$
$4x \geq 10$
Divide both sides by 4, we get
$4x/4 \geq 10/4$
$x \geq 5/2$
∴ The solution of the given inequation is $(5/2, \infty)$.

7. $2(3 - x) \geq x/5 + 4$
Solution:

Given: 2 (3 - x) ≥ x/5 + 4
6 – 2x ≥ x/5 + 4
6 – 2x ≥ (x+20)/5
5(6 – 2x) ≥ (x + 20)
30 – 10x ≥ x + 20
30 – 20 ≥ x + 10x
10 ≥11x
11x ≤ 10
Divide both sides by 11, we get
11x/11 ≤ 10/11
x ≤ 10/11
∴ The solution of the given inequation is (-∞, 10/11).

8. (3x - 2)/5 ≤ (4x - 3)/2
Solution:
Given:
(3x - 2)/5 ≤ (4x - 3)/2
Multiply both the sides by 5 we get,
(3x - 2)/5 × 5 ≤ (4x - 3)/2 × 5
(3x - 2) ≤ 5(4x - 3)/2
3x – 2 ≤ (20x - 15)/2
Multiply both the sides by 2 we get,
(3x - 2) × 2 ≤ (20x - 15)/2 × 2
6x – 4 ≤ 20x – 15
20x – 15 ≥ 6x – 4
20x – 15 + 15 ≥ 6x – 4 + 15
20x ≥ 6x + 11
20x – 6x ≥ 6x + 11 – 6x
14x ≥ 11
Divide both sides by 14, we get
14x/14 ≥ 11/14
x ≥ 11/14
∴ The solution of the given inequation is (11/14, ∞).

9. –(x – 3) + 4 < 5 – 2x
Solution:
Given: –(x – 3) + 4 < 5 – 2x
–x + 3 + 4 < 5 – 2x
–x + 7 < 5 – 2x

$-x + 7 - 7 < 5 - 2x - 7$
$-x < -2x - 2$
$-x + 2x < -2x - 2 + 2x$
$x < -2$
∴ The solution of the given inequation is $(-\infty, -2)$.

10. $x/5 < (3x-2)/4 - (5x-3)/5$
Solution:
Given: $x/5 < (3x-2)/4 - (5x-3)/5$
$x/5 < [5(3x-2) - 4(5x-3)]/4(5)$
$x/5 < [15x - 10 - 20x + 12]/20$
$x/5 < [2 - 5x]/20$
Multiply both the sides by 20 we get,
$x/5 \times 20 < [2 - 5x]/20 \times 20$
$4x < 2 - 5x$
$4x + 5x < 2 - 5x + 5x$
$9x < 2$
Divide both sides by 9, we get
$9x/9 < 2/9$
$x < 2/9$
∴ The solution of the given inequation is $(-\infty, 2/9)$.

11. $[2(x-1)]/5 \leq [3(2+x)]/7$
Solution:
Given:
$[2(x-1)]/5 \leq [3(2+x)]/7$
$(2x - 2)/5 \leq (6 + 3x)/7$
Multiply both the sides by 5 we get,
$(2x - 2)/5 \times 5 \leq (6 + 3x)/7 \times 5$
$2x - 2 \leq 5(6 + 3x)/7$
$7 (2x - 2) \leq 5 (6 + 3x)$
$14x - 14 \leq 30 + 15x$
$14x - 14 + 14 \leq 30 + 15x + 14$
$14x \leq 44 + 15x$
$14x - 44 \leq 44 + 15x - 44$
$14x - 44 \leq 15x$
$15x \geq 14x - 44$
$15x - 14x \geq 14x - 44 - 14x$
$x \geq -44$

∴ The solution of the given inequation is [–44, ∞).

12. 5x/2 + 3x/4 ≥ 39/4

Solution:

Given:

5x/2 + 3x/4 ≥ 39/4

By taking LCM

[2(5x)+3x]/4 ≥ 39/4

13x/4 ≥ 39/4

Multiply both the sides by 4 we get,

13x/4 × 4 ≥ 39/4 × 4

13x ≥ 39

Divide both sides by 13, we get

13x/13 ≥ 39/13

x ≥ 39/13

x ≥ 3

∴ The solution of the given inequation is (3, ∞).

13. (x - 1)/3 + 4 < (x - 5)/5 – 2

Solution:

Given:

(x - 1)/3 + 4 < (x - 5)/5 – 2

Subtract both sides by 4 we get,

(x - 1)/3 + 4 - 4 < (x - 5)/5 – 2 – 4

(x - 1)/3 < (x - 5)/5 – 6

(x - 1)/3 < (x – 5 - 30)/5

(x - 1)/3 < (x - 35)/5

Cross multiply we get,

5 (x - 1) < 3 (x - 35)

5x – 5 < 3x – 105

5x – 5 + 5 < 3x – 105 + 5

5x < 3x – 100

5x – 3x < 3x – 100 – 3x

2x < –100

Divide both sides by 2, we get

2x/2 < -100/2

x < -50

∴ The solution of the given inequation is (-∞, -50).

14. $(2x + 3)/4 – 3 < (x - 4)/3 – 2$

Solution:

Given:

$(2x + 3)/4 – 3 < (x - 4)/3 – 2$

Add 3 on both sides we get,

$(2x + 3)/4 – 3 + 3 < (x - 4)/3 – 2 + 3$

$(2x + 3)/4 < (x - 4)/3 + 1$

$(2x + 3)/4 < (x – 4 + 3)/3$

$(2x + 3)/4 < (x - 1)/3$

Cross multiply we get,

$3 (2x + 3) < 4 (x - 1)$

$6x + 9 < 4x – 4$

$6x + 9 – 9 < 4x – 4 – 9$

$6x < 4x – 13$

$6x – 4x < 4x – 13 – 4x$

$2x < –13$

Divide both sides by 2, we get

$2x/2 < -13/2$

$x < -13/2$

∴ The solution of the given inequation is $(-\infty, -13/2)$.