Exercise 1.6

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

(i) 23/8

Solution:

We have, 23/8 and here the denominator is 8.

⇒ $8 = 2^3 \times 5$

We see that the denominator 8 of 23/8 is of the form $2^m \times 5^n$, where m, n are non-negative integers.

Hence, 23/8 has terminating decimal expansion. And, the decimal expansion of 23/8 terminates after three places of decimal.

(ii) 125/441

Solution:

We have, 125/441 and here the denominator is 441.

⇒ $441 = 3^2 \times 7^2$

We see that the denominator 441 of 125/441 is not of the form $2^m \times 5^n$, where m, n are non-negative integers.

Hence, 125/441 has non-terminating repeating decimal expansion.

(iii) 35/50

Solution:

We have, 35/50 and here the denominator is 50.

⇒ $50 = 2 \times 5^2$

We see that the denominator 50 of 35/50 is of the form $2^m \times 5^n$, where m, n are non-negative integers.

Hence, 35/50 has terminating decimal expansion. And, the decimal expansion of 35/50 terminates after two places of decimal.

(iv) 77/210

Solution:

We have, 77/210 and here the denominator is 210.

⇒ $210 = 2 \times 3 \times 5 \times 7$

We see that the denominator 210 of 77/210 is not of the form $2^m \times 5^n$, where m, n are non-negative integers.

Hence, 77/210 has non-terminating repeating decimal expansion.

(v) $129/(2^2 \times 5^7 \times 7^{17})$

Solution:

We have, $129/(2^2 \times 5^7 \times 7^{17})$ and here the denominator is $2^2 \times 5^7 \times 7^{17}$.

Clearly,

We see that the denominator is not of the form $2^m \times 5^n$, where m, n are non-negative integers.

And hence, 125/441 has non-terminating repeating decimal expansion.

(vi) 987/10500

Solution:

We have, 987/10500

But, 987/10500 = 47/500 (reduced form)

And now the denominator is 500.

$\Rightarrow$ $500 = 2^2 \times 5^3$

We see that the denominator 500 of 47/500 is of the form $2^m \times 5^n$, where m, n are non-negative integers.

Hence, 987/10500 has terminating decimal expansion. And, the decimal expansion of 987/10500 terminates after three places of decimal.

2. Write down the decimal expansions of the following rational numbers by writing their denominators in the form of $2^m \times 5^n$, where m, and n, are the non- negative integers.

(i) 3/8

Solution:

The given rational number is 3/8
It's seen that, $8 = 2^3$ is of the form $2^m \times 5^n$, where m = 3 and n = 0.

So, the given number has terminating decimal expansion.

$$\therefore \frac{3\times5^3}{2^3\times5^3} = \frac{3\times125}{(2\times5)^3} = \frac{375}{(10)^3} = \frac{375}{1000} = 0.375$$

(ii) 13/125

Solution:

The given rational number is 13/125.
It's seen that, $125 = 5^3$ is of the form $2^m \times 5''$, where m = 0 and n = 3.

So, the given number has terminating decimal expansion.

$\therefore 13/125 = (13 \times 2^3)/(125 \times 2^3) = 104/1000 = 0.104$

(iv) 7/80

Solution:

The given rational number is 7/80.

It's seen that, 80 = 2^4 x 5 is of the form 2^m x 5^n, where m = 4 and n = 1.

So, the given number has terminating decimal expansion.

$$\therefore \frac{7}{80} = \frac{7\times5^3}{2^4\times5\times5^3} = \frac{7\times125}{(2\times5)^4} = \frac{875}{10^4} = \frac{875}{10000} = 0.0875$$

(v) 14588/625

Solution:

The given rational number is 14588/625.

It's seen that, 625 = 5^4 is of the form 2^m x 5^n, where m = 0 and n = 4.

So, the given number has terminating decimal expansion.

$$\therefore \frac{14588}{625} = \frac{14588\times2^4}{2^4\times5^4} = \frac{14588}{5^4} = 23.3408$$

(vi) 129/(2^2 x 5^7)

Solution:

The given number is 129/(2^2 x 5^7).
It's seen that, 2^2 x 5^7 is of the form 2^m x 5^n, where m = 2 and n = 7.

So, the given number has terminating decimal expansion.

$$\therefore \frac{129}{2^2\times5^7} = \frac{129\times2^5}{2^2\times5^7\times2^5} = \frac{129\times32}{(2\times5)^7} = \frac{4182}{10^7} = \frac{4128}{10000000} = 0.0004182$$

3. Write the denominator of the rational number 257/5000 in the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, write the decimal expansion, without actual division.

Solution:

The denominator of the given rational number is 5000.

⇒ 5000 = 2^3 x 5^4
It's seen that, 2^3 x 5^4 is of the form 2^m x 5^n, where m = 3 and n = 4.

∴ 257/5000 = (257 x 2)/(5000 x 2) = 514/10000 = 0.0514 is its decimal expansion.

4. What can you say about the prime factorization of the denominators of the following rational:

(i) 43.123456789

Solution:

Since 43.123456789 has terminating decimal expansion. Hence, its denominator is of the form 2^m x 5^n, where m, n are non-negative integers.

(ii) $43.\overline{123456789}$

Solution:

Since the given rational has non-terminating decimal expansion. So, its denominator has factors other than 2 or 5.

(iii) $27.\overline{142857}$

Solution:

Since the given rational number has non-terminating decimal expansion. So, its denominator has factors other than 2 or 5.

(iv) 0.120120012000120000....

Solution:

Since 0.120120012000120000…. has non-terminating decimal expansion. So, its denominator has factors other than 2 or 5.

5. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form p/q? Give reasons.

Solution:

Since, 327.7081 has a terminating decimal expansion its denominator should be of the form 2^m x 5^n, where m, n are non-negative integers.

Further,

327.7081 can be expressed as 3277081/10000 = p/q

$\Rightarrow$ q = 10000 = 2^3 x 5^3

Hence, the prime factors of q has only factors of 2 and 5.