$\frac{x-1}{-3}=\frac{y-(-2)}{2}=\frac{z-3}{6}$

$\frac{x-1}{-3}=\frac{y+2}{2}=\frac{z-3}{6}$

The line parallel to the line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$ and passing through the point

(1,-2,3) is given by $\frac{x-1}{-3}=\frac{y+2}{2}=\frac{z-3}{6}$

5. Question

Find the Cartesian equations of the line which passes through the point (-2, 4, -5) and which is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

Answer

Given : A line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

To find : equations of a line parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

and passing through the point (-2, 4, -5).

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then equation of parallel

line passing through the point (p,q,r) is given by $\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

The given line is $\frac{x+3}{3}=\frac{y-4}{-5}=\frac{z+8}{6}$

Here l = 3 , m = -5 , n = 6 and p = -2 , q = 4 , r = -5

The line parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by

$\frac{x-(-2)}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$

$\frac{x+2}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$

The line parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by $\frac{x+2}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$

6. Question

Write the vector equation of a line whose Cartesian equations are $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

Answer

Given : A line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

To find : vector equation of a line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}=\lambda$ then vector equation of the line is given by $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda\,(l\vec{i} + m\vec{j} + n\vec{k})$

Here a = 5 , b = -4 , c = 6 and l = 3 , m = 7 , n = -2

Substituting the above values,we get

$\vec{r} = 5\vec{i} - 4\vec{j} + 6\vec{k} + \lambda\,(3\vec{i} + 7\vec{j} - 2\vec{k})$

The vector equation of a line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$ is given by

$\vec{r} = 5\vec{i} - 4\vec{j} + 6\vec{k} + \lambda\,(3\vec{i} + 7\vec{j} - 2\vec{k})$

7. Question

The Cartesian equations of a line are $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$. Write the vector equation of the line.

Answer

Given : A line $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$.

To find : vector equation of a line $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$.

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}=\lambda$ then vector equation of the line is given by $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda\,(l\vec{i} + m\vec{j} + n\vec{k})$

The given line is $\frac{x-3}{-5}=\frac{y+4}{7}=\frac{z-3}{2}$

Here a = 3 , b = -4 , c = 3 and l = -5 , m = 7 , n = 2

Substituting the above values,we get

$\vec{r} = 3\vec{i} - 4\vec{j} + 3\vec{k} + \lambda\,(-5\vec{i} + 7\vec{j} + 2\vec{k})$

The vector equation of a line is given by $\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$

$\vec{r} = 3\vec{i} - 4\vec{j} + 3\vec{k} + \lambda\,(-5\vec{i} + 7\vec{j} + 2\vec{k})$

8. Question

Write the vector equation of a line passing through the point (1, -1, 2) and parallel to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

Answer

Given : A line $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

To find : vector equation of a line passing through the point (1, -1, 2) and parallel

to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

Formula used : If a line is parallel to $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ and passing through the point (p,q,r) then vector equation of the line is given by $\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda\,(l\vec{i} + m\vec{j} + n\vec{k})$

The given line is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$

Here p = 1 , q = -1 , c = 2 and l = 1 , m = 2 , n = 2

Substituting the above values,we get

$\vec{r} = 1\vec{i} - 1\vec{j} + 2\vec{k} + \lambda\,(1\vec{i} + 2\vec{j} + 2\vec{k})$

The vector equation of a line passing through the point (1, -1, 2) and

parallel to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$. is given by

$\vec{r} = \vec{i} - \vec{j} + 2\vec{k} + \lambda\,(\vec{i} + 2\vec{j} + 2\vec{k})$

9. Question

If P(1, 5, 4) and Q(4, 1, -2) be two given points, find the direction ratios of PQ.

Answer

Given : P(1, 5, 4) and Q(4, 1, -2) be two given points

To find : direction ratios of PQ

Formula used : if $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two given points then direction ratios of PQ is given by $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$

$x_1 = 1$, $y_1 = 5$, $z_1 = 4$ and $x_2 = 4$, $y_2 = 1$, $z_2 = -2$

Direction ratios of PQ is given by $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$

Direction ratios of PQ is given by $4 - 1, 1 - 5, -2 - 4$

Direction ratios of PQ is given by $3, -4, -6$

Direction ratios of PQ is given by $3, -4, -6$

10. Question

The equations of a line are $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$. Find the direction cosines of a line parallel to this line.

Answer

Given : A line $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$.

To find : Direction cosines of the line parallel to $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$.

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}$, $\frac{m}{\sqrt{l^2+m^2+n^2}}$, $\frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$

Parallel lines have same direction ratios and direction cosines

Here l = -2 , m = 2 , n = 1

Direction cosines of the line $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$ is

$$\frac{-2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{1}{\sqrt{(-2)^2+(2)^2+(1)^2}}$$

$$= \frac{-2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{1}{\sqrt{4+4+1}} = \frac{-2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}$$

$$= \frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$$

Direction cosines of the line parallel to the line $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$ is

$\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$

11. Question

The Cartesian equations of a line are $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$. Find its vector equation.

Answer

Given : A line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$.

To find : vector equation of a line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$.

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}=\lambda$ then vector equation of the line is given by $\vec{r}=a\vec{i}+b\vec{j}+c\vec{k}+\lambda\,(l\vec{i}+m\vec{j}+n\vec{k})$

The given line is $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-5}{-1}$

Here a = 1 , b = -2 , c = 5 and l = 2 , m = 3 , n = -1

Substituting the above values,we get

$\vec{r}=1\vec{i}-2\vec{j}+5\vec{k}+\lambda\,(2\vec{i}+3\vec{j}-1\vec{k})$

The vector equation of a line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$. is given by

$\vec{r}=1\vec{i}-2\vec{j}+5\vec{k}+\lambda\,(2\vec{i}+3\vec{j}-1\vec{k})$

12. Question

Find the vector equation of a line passing through the point (1, 2, 3) and parallel to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

Answer

Given : A vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

To find : vector equation of a line passing through the point (1, 2, 3) and parallel

to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

Formula used : If a line is parallel to the vector $(l\vec{i}+m\vec{j}+n\vec{k})$

and passing through the point (p,q,r) then vector equation of the line is given by

$\vec{r}=p\vec{i}+q\vec{j}+r\vec{k}+\lambda\,(l\vec{i}+m\vec{j}+n\vec{k})$

Here p = 1 , q = 2 , c = 3 and l = 3 , m = 2 , n = -2

Substituting the above values,we get

$\vec{r} = 1\vec{i} + 2\vec{j} + 3\vec{k} + \lambda\,(3\vec{i} + 2\vec{j} - 2\vec{k})$

The vector equation of a line passing through the point (1, 2, 3) and

parallel to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.is $\vec{r} = \vec{i} + 2\vec{j} + 3\vec{k} + \lambda\,(3\vec{i} + 2\vec{j} - 2\vec{k})$

13. Question

The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$. Find its Cartesian equation.

Answer

Given : The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$.

To find : Cartesian equation of the line

Formula used : If the vector equation of the line is given by

$\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda\,(l\vec{i} + m\vec{j} + n\vec{k})$ then its Cartesian equation is given by

$\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$.

Here p = 2, q = 1, r = -4 and l = 1,m = -1,n = -1

Cartesian equation is given by

$\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z-(-4)}{-1}$

$\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z+4}{-1}$

Cartesian equation of the line is given by $\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z+4}{-1}$

14. Question

Find the Cartesian equation of a line which passes through the point (-2, 4, -5) and which is parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

Answer

Given : A line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

To find : cartesian equations of a line parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

and passing through the point (-2, 4, -5).

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then equation of parallel

line passing through the point (p,q,r) is given by $\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

The given line is $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

Here l = 3 , m = 5 , n = 6 and p = -2 , q = 4 , r = -5

The line parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by

$\frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6}$

$\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

The line parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

15. Question

Find the Cartesian equation of a line which passes through the point having position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$ and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$.

Answer

Given : A line which passes through the point having position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$

and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$.

To find : cartesian equations of a line

Formula used : If a line which passes through the point having position vector

$p\vec{i} + q\vec{j} + r\vec{k}$ and is in the direction of the vector $l\vec{i} + m\vec{j} + n\vec{k}$ then its Cartesian

equation is given by

$\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

A line which passes through the point having position vector $\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$

and is in the direction of the vector $\left(\hat{i} + 2\hat{j} - \hat{k}\right)$.

Here l = 1 , m = 2 , n = -1 and p = 2 , q = -1 , r = 4

$\frac{x-2}{1} = \frac{y-(-1)}{2} = \frac{z-4}{-1}$

$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$

The Cartesian equation of a line which passes through the point having

position vector $\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$ and is in the direction of the vector $\left(\hat{i} + 2\hat{j} - \hat{k}\right)$. is

$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$

16. Question

Find the angle between the lines $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and

$\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

Answer

Given : the lines $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

To find : angle between the lines

Formula used : If the lines are $a\vec{i} + b\vec{j} + c\vec{k} + \lambda(p\vec{i} + q\vec{j} + r\vec{k})$ and $d\vec{i} + e\vec{j} + f\vec{k} +$

$\lambda(l\vec{i} + m\vec{j} + n\vec{k})$ then the angle between the lines 'θ' is given by

$\theta = \cos^{-1}\frac{pl + qm + rn}{\sqrt{p^2+q^2+r^2}\sqrt{l^2+m^2+n^2}}$

the lines $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

Here p = 3 , q = 2 , r = 6 and l = 1 , m = 2 , n = 2

$\theta = \cos^{-1}\frac{3(1) + 2(2) + 6(2)}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}} = \cos^{-1}\frac{3+4+12}{\sqrt{9+4+36}\sqrt{1+4+4}}$

$\theta = \cos^{-1}\frac{3+4+12}{\sqrt{49}\sqrt{9}} = \cos^{-1}\frac{19}{7\times 3} = \cos^{-1}\frac{19}{21}$

$\theta = \cos^{-1}\frac{19}{21}$

The angle between the lines $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$. is $\cos^{-1}\frac{19}{21}$

17. Question

Find the angle between the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

Answer

Given : the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

To find : angle between the lines

Formula used : If the lines are $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$ and $\frac{x-c}{l} = \frac{y-d}{m} = \frac{z-e}{n}$

then the angle between the lines 'θ' is given by

$\theta = \cos^{-1}\frac{pl + qm + rn}{\sqrt{p^2 + q^2 + r^2}\sqrt{l^2 + m^2 + n^2}}$

The lines are $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

Here p = 3 , q = 5 , r = 4 and l = 1 , m = 1 , n = 2

$\theta = \cos^{-1}\frac{3(1) + 5(1) + 4(2)}{\sqrt{3^2 + 5^2 + 4^2}\sqrt{1^2 + 1^2 + 2^2}} = \cos^{-1}\frac{3 + 5 + 8}{\sqrt{9 + 25 + 16}\sqrt{1 + 1 + 4}}$

$\theta = \cos^{-1}\frac{3 + 5 + 8}{\sqrt{50}\sqrt{6}} = \cos^{-1}\frac{16}{10\sqrt{3}} = \cos^{-1}\frac{8}{5\sqrt{3}}$

$\theta = \cos^{-1}\frac{8\sqrt{3}}{15}$

The angle between the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

is $\cos^{-1}\frac{8\sqrt{3}}{15}$

18. Question

Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are at right angles.

Answer

Given : the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

To prove : the lines are at right angles.

Formula used : If the lines are $\frac{x-a}{p}=\frac{y-b}{q}=\frac{z-c}{r}$ and $\frac{x-c}{l}=\frac{y-d}{m}=\frac{z-e}{n}$

then the angle between the lines 'θ' is given by

$\theta = \cos^{-1}\frac{pl+qm+rn}{\sqrt{p^2+q^2+r^2}\sqrt{l^2+m^2+n^2}}$

The lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

Here p = 7 , q = -5 , r = 1 and l = 1 , m = 2 , n = 3

$\theta = \cos^{-1}\frac{7(1)+(-5)(2)+1(3)}{\sqrt{7^2+(-5)^2+1}\sqrt{1^2+2^2+3^2}} = \cos^{-1}\frac{7-10+3}{\sqrt{49+25+1}\sqrt{1+4+9}}$

$\theta = \cos^{-1}\frac{0}{\sqrt{75}\sqrt{14}} = \cos^{-1}0 = 90°$

$\theta = 90°$

The Lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are at right angles.

19. Question

The direction ratios of a line are 2, 6, -9. What are its direction cosines?

Answer

Given : A line has direction ratios 2, 6 , -9

To find : Direction cosines of the line

Formula used : If (l,m,n) are the direction ratios of a given line then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

Here l = 2 , m = 6 , n = -9

Direction cosines of the line with direction ratios 2, 6 , -9 is

$\frac{2}{\sqrt{2^2+6^2+(-9)^2}}, \frac{6}{\sqrt{2^2+6^2+(-9)^2}}, \frac{-9}{\sqrt{2^2+6^2+(-9)^2}}$

$= \frac{2}{\sqrt{4+36+81}}, \frac{6}{\sqrt{4+36+81}}, \frac{-9}{\sqrt{4+36+81}} = \frac{2}{\sqrt{121}}, \frac{6}{\sqrt{121}}, \frac{-9}{\sqrt{121}}$

$= \frac{2}{11}, \frac{6}{11}, \frac{-9}{11}$

Direction cosines of the line with direction ratios 2, 6, -9 is $\frac{2}{11}, \frac{6}{11}, \frac{-9}{11}$

20. Question

A line makes angles 90°, 135° and 45° with the positive directions of x-axis, y-axis and z-axis respectively. what are the direction cosines of the line?

Answer

Given : A line makes angles 90°, 135° and 45° with the positive directions of x-axis, y-axis and z-axis respectively.

To find : Direction cosines of the line

Formula used : If a line makes angles α°, β° and γ° with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by $\cos\alpha$, $\cos(180° - \beta)$, $\cos(180° - \gamma)$

α = 90°, β = 135° and γ = 45°

Direction cosines of the line is

$\cos 90°$, $\cos(180° - 135°)$, $\cos(180° - 45°)$

$\cos 90°$, $\cos 45°$, $\cos(135°)$

0, $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Direction cosines of the line is 0, $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

21. Question

What are the direction cosines of the y-axis?

Answer

To find : Direction cosines of the y- axis

Formula used : If a line makes angles α°, β° and γ° with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by $\cos\alpha$, $\cos\beta$, $\cos\gamma$

y-axis makes 90 ° with the x and z axes

α = 90°, β = 0 ° and γ = 90 °

Direction cosines of the line is

$\cos 90°$, $\cos 0°$, $\cos 90°$

0, 1, 0

Direction cosines of the line is 0, 1 , 0

22. Question

What are the direction cosines of the vector $\left(2\hat{i}+\hat{j}-2\hat{k}\right)$?

Answer

Given : A vector $\left(2\hat{i}+\hat{j}-2\hat{k}\right)$?

To find : Direction cosines of the vector

Formula used : If a vector is $l\vec{i} + m\vec{j} + n\vec{k}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}$, $\frac{m}{\sqrt{l^2+m^2+n^2}}$, $\frac{n}{\sqrt{l^2+m^2+n^2}}$

Here l = 2 , m = 1 , n = -2

Direction cosines of the line with direction ratios 2, 1, -2 is

$\frac{2}{\sqrt{2^2+(1)^2+(-2)^2}}, \frac{1}{\sqrt{2^2+(1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(1)^2+(-2)^2}}$

$= \frac{2}{\sqrt{4+1+4}}, \frac{1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} = \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}, \frac{-2}{\sqrt{9}}$

$= \frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$

Direction cosines of the vector is $\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$

23. Question

What is the angle between the vector $\vec{r}=\left(4\hat{i}+8\hat{j}+\hat{k}\right)$ and the x-axis?

Answer

Given : the vector $\vec{r}=\left(4\hat{i}+8\hat{j}+\hat{k}\right)$

To find : angle between the vector and the x-axis

Formula used : If the vector $l\vec{i} + m\vec{j} + n\vec{k}$ and x-axis then the angle between the

lines 'θ' is given by

$\theta = \cos^{-1}\frac{l}{\sqrt{l^2+m^2+n^2}}$

Here l = 4 , m = 8 , n = 1

$\theta = \cos^{-1}\frac{4}{\sqrt{4^2+8^2+1^2}} = \cos^{-1}\frac{4}{\sqrt{16+64+1}}$

$\theta = \cos^{-1}\frac{4}{\sqrt{81}} = \cos^{-1}\frac{4}{9}$

$\theta = \cos^{-1}\frac{4}{9}$

The angle between the vector and the x-axis is $\cos^{-1}\frac{4}{9}$

Objective Questions

1. Question

The direction ratios of two lines are 3, 2, -6 and 1, 2, 2, respectively. The acute angle between these lines is

A. $\cos^{-1}\left(\frac{5}{18}\right)$

B. $\cos^{-1}\left(\frac{3}{20}\right)$

C. $\cos^{-1}\left(\frac{5}{21}\right)$

D. $\cos^{-1}\left(\frac{8}{21}\right)$

Answer

Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be $|\vec{a}| = 3\hat{i} + 2\hat{j} - 6\hat{k}$ and second parallel vector be $|\vec{b}| = \hat{i} + 2\,\hat{j} + 2\hat{k}$.

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{3^2 + 2^2 + (-6)^2}$

$= 7$

$|\vec{b}| = \sqrt{1 + 2^2 + 2^2}$

$= 3$

$$\cos\alpha = \frac{(3\hat{i} + 2\hat{j} - 6\hat{k}).(\hat{i} + 2\,\hat{j} + 2\hat{k})}{7 \times 3}$$

$$\cos\alpha = \frac{3 + 4 - 12}{21}$$

$\cos\alpha = \frac{-5}{21}$

$\alpha = \cos^{-1}\left(-\frac{5}{21}\right)$

The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant.

$\alpha = \cos^{-1}\left(\frac{5}{21}\right)$

2. Question

The direction ratios of two lines are a, b, c and (b – c), (c – a), (a – b) respectively. The angle between these lines is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

Answer

Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be $|\vec{a}| = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ and second parallel vector be $|\vec{b}| = (b - c)\hat{\imath} + (c - a)\,\hat{\jmath} + (a - b)\hat{k}.$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{a^2 + b^2 + (c)^2}$

$= \sqrt{a^2 + b^2 + c^2}$

$|\vec{b}| = \sqrt{(b - c)^2 + (c - a)^2 + (a - b)^2}$

$= \sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)}$

$\cos\alpha = \frac{\left(a\hat{\imath} + b\hat{\jmath} + c\hat{k}\right).\left((b - c)\hat{\imath} + (c - a)\,\hat{\jmath} + (a - b)\hat{k}\right)}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$

$$\cos\alpha = \frac{ab - ac + bc - ba + ca - cb}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\cos\alpha = \frac{0}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\alpha = \cos^{-1}(0)$$

$$\alpha = \frac{\pi}{2}$$

3. Question

The angle between the lines $\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\cos^{-1}\left(\frac{3}{8}\right)$

Answer

Direction ratio are given implies that we can write the parallel vector towards those line, lets consider first parallel vector to be $|\vec{a}| = 2\hat{i} + 7\hat{j} - 3\hat{k}$ and second parallel vector be $|\vec{b}| = -\hat{i} + 2\,\hat{j} + 4\hat{k}$.

For the angle we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that we need to find magnitude of these vectors

$$|\vec{a}| = \sqrt{3^2 + 2^2 + (7)^2}$$

$$= \sqrt{62}$$

$$|\vec{b}| = \sqrt{1 + 2^2 + 4^2}$$

$$= \sqrt{21}$$

$$\cos\alpha = \frac{(2\hat{i} + 7\hat{j} - 3\hat{k}).(-\hat{i} + 2\hat{j} + 4\hat{k})}{\sqrt{21} \times \sqrt{62}}$$

$$\cos\alpha = \frac{-2 + 14 - 12}{\sqrt{21} \times \sqrt{62}}$$

$$\cos\alpha = \frac{0}{\sqrt{21} \times \sqrt{62}}$$

$$\alpha = \cos^{-1} 0$$

Negative sign does not affect anything in cosine as cosine is positive in fourth quadrant

$$\alpha = \frac{\pi}{2}$$

4. Question

If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other then k = ?

A. $\frac{-5}{7}$

B. $\frac{5}{7}$

C. $\frac{10}{7}$

D. $\frac{-10}{7}$

Answer

If the lines are perpendicular to each other then the angle between these lines will be

$\frac{\pi}{2}$, me the cosine will be 0

$$\vec{a} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$$

$$|\vec{a}| = \sqrt{3^2 + (2k)^2 + 2^2}$$

$$= \sqrt{13 + 4k^2}$$

$$\vec{b} = 3k\hat{i} + \hat{j} - 5\hat{k}$$

$|\vec{b}| = \sqrt{(3k)^2 + 1 + 5^2}$

$= \sqrt{9k^2 + 26}$

$$\cos\left(\frac{\pi}{2}\right) = \frac{(3k\hat{i} + \hat{j} - 5\hat{k}).(-3\hat{i} + 2k\hat{j} + 2\hat{k})}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$$

$$0 = \frac{-9k + 2k - 10}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$$

$$k = -\frac{10}{7}$$

5. Question

A line passes through the points A(2, -1, 4) and B(1, 2, -2). The equations of the line AB are

A. $\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6}$

B. $\frac{x+2}{-1} = \frac{y+1}{2} = \frac{z-4}{6}$

C. $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{6}$

D. none of these

Answer

To write the equation of a line we need a parallel vector and a fixed point through which the line is passing

Parallel vector= $((2-1)\hat{i} + (-1-2)\hat{j} + (4+2)\hat{k})$

$= \hat{i} - 3\hat{j} + 6\hat{k}$

Or = $-(\hat{i} - 3\hat{j} + 6\hat{k})$

Fixed point is $2\hat{i} - \hat{j} + 4\hat{k}$

Equation

$$\frac{x-2}{1} = \frac{y-(-1)}{-3} = \frac{z-4}{6}$$

$$\frac{x-2}{1} = \frac{y+1}{-3} = \frac{z-4}{6}$$

Or

$$\frac{x-2}{-1}=\frac{y+1}{3}=\frac{z-4}{-6}$$

6. Question

The angle between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ is

A. $\cos^{-1}\left(\frac{3}{4}\right)$

B. $\cos^{-1}\left(\frac{5}{6}\right)$

C. $\cos^{-1}\left(\frac{2}{3}\right)$

D. $\frac{\pi}{3}$

Answer

Direction cosine of the lines are given $2\hat{i}+2\hat{j}+\hat{k}$ and $4\hat{i}+\hat{j}+8\hat{k}$

$\vec{a}=2\hat{i}+2\hat{j}+\hat{k}$

$|\vec{a}|=\sqrt{2^2+2^2+1}$

$|\vec{a}|=3$

$\vec{b}=4\hat{i}+\hat{j}+8\hat{k}$

$|\vec{b}|=\sqrt{4^2+1+8^2}$

$=9$

$$\cos\alpha=\frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$$

$$\cos\alpha=\frac{(2\hat{i}+2\hat{j}+\hat{k}).(4\hat{i}+\hat{j}+8\hat{k})}{3\times 9}$$

$$\cos\alpha=\frac{8+8+2}{27}$$

$$\cos\alpha=\frac{2}{3}$$

7. Question

The angle between the lines $\vec{r}=\left(3\hat{i}+\hat{j}-2\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-2\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}-\hat{j}-5\hat{k}\right)+\mu\left(3\hat{i}-5\hat{j}-4\hat{k}\right)$ is

A. $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

B. $\cos^{-1}\left(\frac{6\sqrt{2}}{5}\right)$

C. $\cos^{-1}\left(\frac{5\sqrt{3}}{8}\right)$

D. $\cos^{-1}\left(\frac{5\sqrt{2}}{6}\right)$

Answer

Let $\vec{a}=\hat{\imath}-\hat{\jmath}-2\hat{k}$ and $\vec{b}=3\hat{\imath}-5\hat{\jmath}-4\hat{k}$ and $|\vec{a}|=\sqrt{1+1+2^2}=\sqrt{6}$

$|\vec{b}|=\sqrt{3^2+5^2+4^2}=5\sqrt{2}$

$\cos\alpha=\frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

$\cos\alpha=\frac{(3\hat{\imath}-5\hat{\jmath}-4\hat{k}).(\hat{\imath}-\hat{\jmath}-2\hat{k})}{5\sqrt{2}\times\sqrt{6}}$

$\cos\alpha=\frac{3+5+8}{5\sqrt{12}}$

$\cos\alpha=\frac{8\sqrt{3}}{15}$

8. Question

A line is perpendicular to two lines having direction ratios 1, -2, -2 and 0, 2, 1. The direction cosines of the line are

A. $\frac{-2}{3},\frac{1}{3},\frac{2}{3}$

B. $\frac{2}{3},\frac{1}{3},\frac{-1}{3}$

C. $\frac{2}{3},\frac{-1}{3},\frac{2}{3}$

D. none of these

Answer

If a line is perpendicular to two given lines we can find out the parallel vector by cross product of the given two vectors.

$\vec{a} = \hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

$\vec{b} = 2\hat{\jmath} + \hat{k}$

$\vec{a} \times \vec{b} = (\hat{\imath} - 2\hat{\jmath} - 2\hat{k}) \times (2\hat{\jmath} + \hat{k})$

$= 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

So the direction cosines are

$\hat{n} = \frac{1}{\sqrt{2^2 + 1 + 2^2}}$

$\hat{n} = \frac{1}{3}$

Direction cosine

$\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

9. Question

A line passes through the point A(5, -2, 4) and it is parallel to the vector $\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$. The vector equation of the line is

A. $\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(5\hat{i} - 2\hat{j} + 4\hat{k}\right)$

B. $\vec{r} = \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

C. $\vec{r} \cdot \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) = \sqrt{14}$

D. none of these

Answer

Fixed point is $5\hat{i}-2\hat{j}+4\hat{k}$ and parallel vector is $2\hat{i}-\hat{j}+3\hat{k}$

Equation $5\hat{i}-2\hat{j}+4\hat{k}+\alpha(2\hat{i}-\hat{j}+3\hat{k})$

10. Question

The Cartesian equations of a line are $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-5}{-1}$. Its vector equation is

A. $\vec{r}=\left(-\hat{i}+2\hat{j}-5\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-\hat{k}\right)$

B. $\vec{r}=\left(2\hat{i}+3\hat{j}-\hat{k}\right)+\lambda\left(\hat{i}-2\hat{j}+5\hat{k}\right)$

C. $\vec{r}=\left(\hat{i}-2\hat{j}+5\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-4\hat{k}\right)$

D. none of these

Answer

Dixed point (1,-2,5) and the parallel vector is $2\hat{i}+3\hat{j}-\hat{k}$

Equation (î -2ĵ +5k ̂)+α $(2\hat{i}+3\hat{j}-\hat{k})$

11. Question

A line passes through the pointA(-2, 4, -5) and is parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$. The vector equation of the line is

A. $\vec{r}=\left(-3\hat{i}+4\hat{j}-8\hat{k}\right)+\lambda\left(-2\hat{i}+4\hat{j}-5\hat{k}\right)$

B. $r=(-2\hat{i}+4\hat{j}-5\hat{k})+\lambda(3\hat{i}+5\hat{j}+6\hat{k})$

C. $\vec{r}=\left(3\hat{i}+5\hat{j}+6\hat{k}\right)+\lambda\left(-2\hat{i}+4\hat{j}-5\hat{k}\right)$

D. none of these

Answer

Fixed point is $-2\hat{i}+4\hat{j}-5\hat{k}$ and the parallel vector is $3\hat{i}+5\hat{j}+6\hat{k}$

Equation is $r=(-2\hat{i}+4\hat{j}-5\hat{k})+\lambda(3\hat{i}+5\hat{j}+6\hat{k})$

12. Question

The coordinates of the point where the line through the points A(5, 1, 6) and B(3, 4, 1) crosses the yz-plane is

A. (0, 17, -13)

B. $\left(0, \frac{-17}{2}, \frac{13}{2}\right)$

C. $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

D. none of these

Answer

We first need to find the equation of a line passing through the two given points

taking fixed point as $5\hat{i} + \hat{j} + 6\hat{k}$

and the parallel vector will be $(5-3)\hat{i} + (1-4)\hat{j} + (6-1)\hat{k} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

equation of the line in cartesian form

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5}$$

Assume above equation to be equal to k, a constant

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = k$$

And y-z plane have x-coordinate as zero we may get

$$\frac{0-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = k$$

$$k = -\frac{5}{2}$$

Now we can find y and z

$$\frac{y-1}{-3} = -\frac{5}{2}$$

$$y - 1 = \frac{15}{2}$$

$$y = \frac{17}{2}$$

$$\frac{z-6}{5} = -\frac{5}{2}$$

$$z - 6 = -\frac{25}{2}$$

$z = -\frac{13}{2}$

The coordinate where the line meets y-z plane is $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$

13. Question

The vector equation of the x-axis is given by

A. $\vec{r} = \hat{i}$

B. $\vec{r} = \hat{j} + \hat{k}$

C. $\vec{r} = \lambda\hat{i}$

D. none of these

Answer

Vector equation need a fixed point and a parallel vector

For x-axis fixed point can be anything ranging from negative to positive including origin

And parallel vector is $\hat{i}$

Equation would be $\lambda\, \hat{i}$

14. Question

The Cartesian equations of a lines are $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}$. What is its vector equation?

A. $\vec{r} = \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

B. $\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

C. $\vec{r} = \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

D. none of these

Answer

Fixed point is $2\hat{i} - \hat{j} + 3\hat{k}$ and the vector is $2\hat{i} + 3\hat{j} - 2\hat{k}$

Equation $\left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

15. Question

The angle between two lines having direction ratios 1, 1, 2 and $\left(\sqrt{3}-1\right)$, $\left(-\sqrt{3}-1\right)$, 4 is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = (\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k}$

$|\vec{a}| = \sqrt{6}\ |\vec{b}| = \sqrt{(4 - 2\sqrt{3}) + (4 + 2\sqrt{3}) + 16} = 2\sqrt{6}$

$$\cos\alpha = \frac{(\hat{i} + \hat{j} + 2\hat{k}).\left((\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k}\right)}{\sqrt{6} \times 2\sqrt{6}}$$

$$\cos\alpha = \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{12}$$

$$\cos\alpha = \frac{1}{2}$$

$\alpha = 60°$

16. Question

The straight line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{0}$ is

A. parallel to the x-axis

B. parallel to the y-axis

C. parallel to the z-axis

D. perpendicular to the z-axis

Answer

It is perpendicular to z-axis because cos90° is 0 which implies that it makes 90° with z-axis

17. Question

If a line makes angles α, β and γ with the x-axis, y-axis and z-axis respectively then ($\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$) = ?

A. 1

B. 3

C. 2

D. $\frac{3}{2}$

Answer

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1 - \cos^2\alpha + 1 - \cos\beta + 1 - \cos^2\gamma$$

$$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

$(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$ is the square of the direction ratios of all three axes which is always equal to 1

$$= 3 - 1$$

$$= 2$$

18. Question

If (a_1, b_1, c_1) and (a_2, b_2, c_2) be the direction ratios of two parallel lines then

A. $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$

B. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

C. $a_1^2 + b_1^2 + c_1^2 = a_2^2 + b_2^2 + c_2^2$

D. $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Answer

We know that if there is two parallel lines then their direction ratios must have a relation

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

19. Question

If the points A(-1, 3, 2), B(-4, 2, -2) and C(5, 5, λ) are collinear then the value of λ is

A. 5

B. 7

C. 8

D. 10

Answer

Determinant of these point should be zero

$$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$$

$$-1(2\lambda + 10) - 3(-4\lambda + 10) + 2(-20 - 10) = 0$$

$$10\lambda - 10 - 30 - 60 = 0$$

$$\lambda = 10$$