EXERCISE 5.2

1. Compute the following sums:

(i) $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$

Solution:

(i) Given

$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$

Corresponding elements of two matrices should be added

Therefore, we get

$= \begin{bmatrix} 3-2 & -2+4 \\ 1+1 & 4+3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$

Therefore, $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$

(ii) Given

$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2+1 & 1-2 & 3+3 \\ 0+2 & 3+6 & 5+1 \\ -1+0 & 2-3 & 5+1 \end{bmatrix}$

$= \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$

Therefore,

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

$$2. Let\ A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\ B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\ and\ C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$$

Find each of the following:

(i) 2A – 3B

(ii) B – 4C

(iii) 3A – C

(iv) 3A – 2B + 3C

Solution:

(i) Given

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\ B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\ and\ C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$$

First we have to compute 2A

$$2A = 2\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix}$$

Now by computing 3B we get,

$$= 3B = 3\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}$$

Now by we have to compute 2A – 3B we get

$$= 2A - 3B = \begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix} = \begin{bmatrix} 4-3 & 8-9 \\ 6+6 & 4-15 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

Therefore

$$2A - 3B = \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

(ii) Given $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\ B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\ and\ C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$

First we have to compute 4C,

$$4C=4\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}=\begin{bmatrix}-8 & 20\\ 12 & 16\end{bmatrix}$$

Now,

$$B-4C=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}-\begin{bmatrix}-8 & 20\\ 12 & 16\end{bmatrix}$$

$$=\begin{bmatrix}1+8 & 3-20\\ -2-12 & 5-16\end{bmatrix}=\begin{bmatrix}9 & -17\\ -14 & -11\end{bmatrix}$$

Therefore we get,

$$B-4C=\begin{bmatrix}9 & -17\\ -14 & -11\end{bmatrix}$$

(iii) Given

$$A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}\ B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}\ and\ C=\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}.$$

First we have to compute 3A,

$$3A=3\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}=\begin{bmatrix}6 & 12\\ 9 & 6\end{bmatrix}$$

Now,

$$=3A-C=\begin{bmatrix}6 & 12\\ 9 & 6\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}6+2 & 12-5\\ 9-3 & 6-4\end{bmatrix}=\begin{bmatrix}8 & 7\\ 6 & 2\end{bmatrix}$$

Therefore,

$$3A-C=\begin{bmatrix}8 & 7\\ 6 & 2\end{bmatrix}$$

(iv) Given

$$A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}\ B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}\ and\ C=\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}.$$

First we have to compute 3A

$3A=3\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}=\begin{bmatrix}6 & 12\\9 & 6\end{bmatrix}$

Now we have to compute 2B

$= 2B=2\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}=\begin{bmatrix}2 & 6\\-4 & 10\end{bmatrix}$

By computing 3C we get,

$= 3C=3\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}=\begin{bmatrix}-6 & 15\\9 & 12\end{bmatrix}$

$= 3A-2B+3C=\begin{bmatrix}6 & 12\\9 & 6\end{bmatrix}-\begin{bmatrix}2 & 6\\-4 & 10\end{bmatrix}+\begin{bmatrix}-6 & 15\\9 & 12\end{bmatrix}$

$=\begin{bmatrix}6-2-6 & 12-6+15\\9+4+9 & 6-10+12\end{bmatrix}=\begin{bmatrix}-2 & 21\\22 & 8\end{bmatrix}$

Therefore,

$3A-2B+3C=\begin{bmatrix}-2 & 21\\22 & 8\end{bmatrix}$

$3. If\ A=\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}, B=\begin{bmatrix}-1 & 0 & 2\\3 & 4 & 1\end{bmatrix}, C=\begin{bmatrix}-1 & 2 & 3\\2 & 1 & 0\end{bmatrix}, find$

(i) A + B and B + C

(ii) 2B + 3A and 3C – 4B

Solution:

(i) Consider A + B,

A + B is not possible because matrix A is an order of 2 x 2 and Matrix B is an order of 2 x 3, so the Sum of the matrix is only possible when their order is same.

Now consider B + C

$\Rightarrow B+C=\begin{bmatrix}-1 & 0 & 2\\3 & 4 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2 & 3\\2 & 1 & 0\end{bmatrix}$

$\Rightarrow B+C=\begin{bmatrix}-1-1 & 0+2 & 2+3\\3+2 & 4+1 & 1+0\end{bmatrix}$

$\Rightarrow B+C=\begin{bmatrix}-2 & 2 & 5\\5 & 5 & 1\end{bmatrix}$

(ii) Consider 2B + 3A

2B + 3A also does not exist because the order of matrix B and matrix A is different, so we cannot find the sum of these matrix.

Now consider 3C – 4B,

$$\Rightarrow 3C-4B=3\begin{bmatrix}-1 & 2 & 3\\ 2 & 1 & 0\end{bmatrix}-4\begin{bmatrix}-1 & 0 & 2\\ 3 & 4 & 1\end{bmatrix}$$

$$\Rightarrow 3C-4B=\begin{bmatrix}-3 & 6 & 9\\ 6 & 3 & 0\end{bmatrix}-\begin{bmatrix}-4 & 0 & 8\\ 12 & 16 & 4\end{bmatrix}$$

$$\Rightarrow 3C-4B=\begin{bmatrix}-3+4 & 6-0 & 9-8\\ 6-12 & 3-16 & 0-4\end{bmatrix}$$

$$\Rightarrow 3C-4B=\begin{bmatrix}1 & 6 & 1\\ -6 & -13 & -4\end{bmatrix}$$

4. Let $A=\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & 4\end{bmatrix}, B=\begin{bmatrix}0 & -2 & 5\\ 1 & -3 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1 & -5 & 2\\ 6 & 0 & -4\end{bmatrix}$. Compute $2A-3B+4C$

Solution:

Given

$$A=\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & 4\end{bmatrix}, B=\begin{bmatrix}0 & -2 & 5\\ 1 & -3 & 1\end{bmatrix} \text{ and } C=\begin{bmatrix}1 & -5 & 2\\ 6 & 0 & -4\end{bmatrix}$$

Now we have to compute 2A – 3B + 4C

$$2A-3B+4C=2\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & 4\end{bmatrix}-3\begin{bmatrix}0 & -2 & 5\\ 1 & -3 & 1\end{bmatrix}+4\begin{bmatrix}1 & -5 & 2\\ 6 & 0 & -4\end{bmatrix}$$

$$\Rightarrow 2A-3B+4C=\begin{bmatrix}-2 & 0 & 4\\ 6 & 2 & 8\end{bmatrix}-\begin{bmatrix}0 & -6 & 15\\ 3 & -9 & 3\end{bmatrix}+\begin{bmatrix}4 & -20 & 8\\ 24 & 0 & -16\end{bmatrix}$$

$$\Rightarrow 2A - 3B + 4C = \begin{bmatrix} -2-0+4 & 0+6-20 & 4-15+8 \\ 6-3+24 & 2+9+0 & 8-3-16 \end{bmatrix}$$

$$\Rightarrow 2A - 3B + 4C = \begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix}$$

5. If A = diag (2 -5 9), B = diag (1 1 -4) and C = diag (-6 3 4), find
(i) A – 2B
(ii) B + C - 2A
(iii) 2A + 3B – 5C

Solution:
(i) Given A = diag (2 -5 9), B = diag (1 1 -4) and C = diag (-6 3 4)

Here,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$A - 2B$

$$\Rightarrow A-2B= \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} -2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow A-2B= \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

$$\Rightarrow A-2B= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 17 \end{bmatrix} = \text{diag } (0\ -7\ 17)$$

(ii) Given A = diag (2 -5 9), B = diag (1 1 -4) and C = diag (-6 3 4)
We have to find B + C - 2A

Here,

A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$ C = $\begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Now we have to compute B + C - 2A

$$\Rightarrow B + C - 2A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} - 2\begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow B + C - 2A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

$$\Rightarrow B + C - 2A = \begin{bmatrix} 1-6-4 & 0+0-0 & 0+0-0 \\ 0+0-0 & 1+3+10 & 0+0-0 \\ 0+0-0 & 0+0-0 & -4+4-18 \end{bmatrix}$$

$$\Rightarrow B + C - 2A = \begin{bmatrix} -9 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & -18 \end{bmatrix} = diag(-9\ 14\ -18)$$

(iii) Given A = diag (2 -5 9), B = diag (1 1 -4) and C = diag (-6 3 4)

Now we have to find 2A + 3B – 5C

Here,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

and C = $\begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Now consider 2A + 3B – 5C

$$\Rightarrow 2A+3B-5C=2\begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}+3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}-5\begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow 2A+3B-5C=\begin{bmatrix} 4 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 18 \end{bmatrix}+\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -12 \end{bmatrix}-\begin{bmatrix} -30 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$\Rightarrow 2A+3B-5C=\begin{bmatrix} 4+3+30 & 0+0-0 & 0+0-0 \\ 0+0-0 & -10+3-15 & 0+0-0 \\ 0+0-0 & 0+0-0 & 18-12-20 \end{bmatrix}$$

$$\Rightarrow 2A+3B-5C=\begin{bmatrix} 37 & 0 & 0 \\ 0 & -22 & 0 \\ 0 & 0 & -14 \end{bmatrix}$$

$=\text{diag}(37\ -22\ -14)$

6. Given the matrices

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B=\begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \text{ and } C=\begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

Verify that (A + B) + C = A + (B + C)

Solution:

Given

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B=\begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \text{ and } C=\begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

Now we have to verify (A + B) + C = A + (B + C)

First consider LHS, (A + B) + C,

$$=\left(\begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}+\begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix}\right)+\begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$=\left(\begin{bmatrix} 2+9 & 1+7 & 1-1 \\ 3+3 & -1+5 & 0+4 \\ 0+2 & 2+1 & 4+6 \end{bmatrix}\right)+\begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 8 & 0 \\ 6 & 4 & 4 \\ 2 & 3 & 10 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11+2 & 8-4 & 0+3 \\ 6+1 & 4-1 & 4+0 \\ 2+9 & 3+4 & 10+5 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix}$$

Now consider RHS, that is A + (B + C)

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \left(\begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \left(\begin{bmatrix} 9+2 & 7-4 & -1+3 \\ 3+1 & 5-1 & 4+0 \\ 2+9 & 1+4 & 6+5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 11 & 3 & 2 \\ 4 & 4 & 4 \\ 11 & 5 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 2+11 & 1+3 & 1+2 \\ 3+4 & -1+4 & 0+4 \\ 0+11 & 2+5 & 4+11 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix}$$

Therefore LHS = RHS

Hence (A + B) + C = A + (B + C)

7. Find the matrices X and Y,

$$\textbf{if } \mathbf{X} + \mathbf{Y} = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \textbf{ and } \mathbf{X} - \mathbf{Y} = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

Solution:

Consider,

$$(X+Y)+(X-Y)=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}+\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$$

Now by simplifying we get,

$$\Rightarrow 2X=\begin{bmatrix}5+3 & 2+6\\ 0+0 & 9-1\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}8 & 8\\ 0 & 8\end{bmatrix}$$

$$\Rightarrow X=\frac{1}{2}\begin{bmatrix}8 & 8\\ 0 & 8\end{bmatrix}$$

Therefore,

$$\Rightarrow X=\begin{bmatrix}4 & 4\\ 0 & 4\end{bmatrix}$$

Again consider,

$$(X+Y)-(X-Y)=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}-\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$$

$$\Rightarrow X+Y-X+Y=\begin{bmatrix}5-3 & 2-6\\ 0-0 & 9+1\end{bmatrix}$$

Now by simplifying we get,

$$\Rightarrow 2Y=\begin{bmatrix}2 & -4\\ 0 & 10\end{bmatrix}$$

$$\Rightarrow Y=\frac{1}{2}\begin{bmatrix}2 & -4\\ 0 & 10\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$$

Therefore,

$$X=\begin{bmatrix}4 & 4\\ 0 & 4\end{bmatrix} \text{ and } Y=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$$

8.Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

Solution:

Given

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

Now by transposing, we get

$$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

Therefore,

$$\Rightarrow X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

9.Find matrices X and Y, if $2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$.

Solution:

Given

$$(2X - Y) = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \dots (1)$$

$$(X + 2Y) = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \dots (2)$$

Now by multiplying equation (1) and (2) we get,

$$2(2X - Y) = 2\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow 4X-2Y=\begin{bmatrix}12 & -12 & 0\\ -8 & 4 & 2\end{bmatrix}\ldots(3)$$

Now by adding equation (2) and (3) we get,

$$(4X-2Y)+(X+2Y)=\begin{bmatrix}12 & -12 & 0\\ -8 & 4 & 2\end{bmatrix}+\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$$

$$\Rightarrow 5X=\begin{bmatrix}12+3 & -12+2 & 0+5\\ -8-2 & 4+1 & 2-7\end{bmatrix}$$

$$\Rightarrow 5X=\begin{bmatrix}15 & -10 & 5\\ -10 & 5 & -5\end{bmatrix}$$

$$\Rightarrow X=\frac{1}{5}\begin{bmatrix}15 & -10 & 5\\ -10 & 5 & -5\end{bmatrix}$$

$$\Rightarrow X=\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$$

Now by substituting X in equation (2) we get,

$$(X+2Y)=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}+2Y=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$$

$$\Rightarrow 2Y=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}-\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$$

$$\Rightarrow 2Y=\begin{bmatrix}3-3 & 2+2 & 5-1\\ -2+2 & 1-1 & -7+1\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$$

10. If $X-Y=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$ and $X+Y=\begin{bmatrix}3 & 5 & 1\\ -1 & 1 & 4\\ 11 & 8 & 0\end{bmatrix}$ find X and Y.

Solution:

Consider

$$X-Y+X+Y=\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}+\begin{bmatrix}3&5&1\\-1&1&4\\11&8&0\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}1+3&1+5&1+1\\1-1&1+1&0+4\\1+11&0+8&0+0\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}4&6&2\\0&2&4\\12&8&0\end{bmatrix}$$

$$\Rightarrow X=\frac{1}{2}\begin{bmatrix}4&6&2\\0&2&4\\12&8&0\end{bmatrix}$$

$$\Rightarrow X=\begin{bmatrix}2&3&1\\0&1&2\\6&4&0\end{bmatrix}$$

Now, again consider

$$(X-Y)-(X+Y)=\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}-\begin{bmatrix}3&5&1\\-1&1&4\\11&8&0\end{bmatrix}$$

$$\Rightarrow X-Y-X-Y=\begin{bmatrix}1-3&1-5&1-1\\1+1&1-1&0-4\\1-11&0-8&0-0\end{bmatrix}$$

$$\Rightarrow -2Y=\begin{bmatrix}-2&-4&0\\2&0&-4\\-10&-8&0\end{bmatrix}$$

$$\Rightarrow Y=-\frac{1}{2}\begin{bmatrix}-2&-4&0\\2&0&-4\\-10&-8&0\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}1&2&0\\-1&0&2\\5&4&0\end{bmatrix}$$

Therefore,

$$X = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

And

$$Y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$