Exercise 8(C)

1. If $\log_{10} 8 = 0.90$; find the value of:
(i) $\log_{10} 4$
(ii) $\log \sqrt{32}$
(iii) $\log 0.125$
Solution:

Given, $\log_{10} 8 = 0.90$
$\log_{10} (2 \times 2 \times 2) = 0.90$
$\log_{10} 2^3 = 0.90$
$3 \log_{10} 2 = 0.90$
$\log_{10} 2 = 0.90/3$
$\log_{10} 2 = 0.30$ … (1)

(i) $\log 4 = \log_{10} (2 \times 2)$
$= \log_{10} 2^2$
$= 2 \log_{10} 2$
$= 2 \times 0.60$ … [From (1)]
$= 1.20$

(ii) $\log \sqrt{32} = \log_{10} 32^{1/2}$
$= ½ \log_{10} (2 \times 2 \times 2 \times 2 \times 2)$
$= ½ \log_{10} 2^5$
$= ½ \times 5 \log_{10} 2$
$= ½ \times 5 \times 0.30$ … [From (1)]
$= 0.75$

(iii) $\log 0.125 = \log 125/1000$
$= \log_{10} 1/8$
$= \log_{10} 1/2^3$
$= \log_{10} 2^{-3}$
$= -3 \log_{10} 2$
$= -3 \times 0.30$ … [From (1)]
$= -0.90$

2. If $\log 27 = 1.431$, find the value of:
(i) $\log 9$ (ii) $\log 300$
Solution:

Given, $\log 27 = 1.431$
So, $\log 3^3 = 1.431$
$3\log 3 = 1.431$
$\log 3 = 1.431/3$
$= 0.477$ … (1)

(i) $\log 9 = \log 3^2$

= 2log 3
= 2 x 0.477 … [From (1)]
= 0.954

(ii) log 300 = log (3 x 100)
= log 3 + log 100
= log 3 + log 10^2
= log 3 + 2log 10
= log 3 + 2 [As log 10 = 1]
= 0.477 + 2
= 2.477

3. If $\log_{10}$ a = b, find 10^{3b-2} in terms of a.
Solution:

Given, $\log_{10}$ a = b
Now,
Let 10^{3b-2} = x
Applying log on both sides,
log 10^{3b-2} = log x
(3b – 2)log 10 = log x
3b – 2 = log x
3$\log_{10}$ a – 2 = log x
3$\log_{10}$ a – 2log 10 = log x
$\log_{10}$ a^3 – log 10^2 = log x
$\log_{10}$ a^3 – log 100 = log x
$\log_{10}$ a^3/100 = log x
On removing logarithm, we get
a^3/100 = x
Hence, 10^{3b-2} = a^3/100

4. If $\log_5$ x = y, find 5^{2y+3} in terms of x.
Solution:

Given, $\log_5$ x = y
So, 5^y = x
Squaring on both sides, we get
$(5^y)^2$ = x^2
5^{2y} = x^2
$5^{2y} \times 5^3$ = $x^2 \times 5^3$
Hence,
5^{2y+3} = 125x^2

5. Given: $\log_3$ m = x and $\log_3$ n = y.
(i) Express 3^{2x-3} in terms of m.
(ii) Write down $3^{1-2y+3x}$ in terms of m and n.

(iii) If 2 $\log_3 A = 5x - 3y$; find A in terms of m and n.

Solution:

Given, $\log_3 m = x$ and $\log_3 n = y$
So, $3^x = m$ and $3^y = n$ … (1)
(i) Taking the given expression, $3^{2x - 3}$

$3^{2x - 3} = 3^{2x} . 3^{-3}$
$= 3^{2x} . 1/3^3$
$= (3^x)^2/3^3$
$= m^2/3^3$ … [Using (1)]
$= m^2/27$
Hence, $3^{2x - 3} = m^2/27$

(ii) Taking the given expression, $3^{1 - 2y + 3x}$

$3^{1 - 2y + 3x} = 3^1 . 3^{-2y} . 3^{3x}$
$= 3 . (3^y)^{-2} . (3^x)^3$
$= 3 . n^{-2} . m^3$ … [Using (1)]
$= 3m^3/n^2$
Hence, $3^{1 - 2y + 3x} = 3m^3/n^2$

(iii) Taking the given equation,
$2 \log_3 A = 5x - 3y$
$\log_3 A^2 = 5x - 3y$
$\log_3 A^2 = 5\log_3 m - 3\log_3 n$ … [Using (1)]
$\log_3 A^2 = \log_3 m^5 - \log_3 n^3$
$\log_3 A^2 = \log_3 m^5/n^3$
Removing logarithm on both sides, we get
$A^2 = m^5/n^3$
Hence, by taking square root on both sides
$A = \sqrt{(m^5/n^3)} = m^{5/2}/n^{3/2}$

6. Simplify:
(i) $\log (a)^3 - \log a$
(ii) $\log (a)^3 \div \log a$
Solution:

(i) We have, $\log (a)^3 - \log a$
$= 3\log a - \log a$
$= 2\log a$

(ii) We have, $\log (a)^3 \div \log a$
$= 3\log a/ \log a$
$= 3$

7. If $\log (a + b) = \log a + \log b$, find a in terms of b.

Solution:

We have, log (a + b) = log a + log b
Then,
log (a + b) = log ab
So, on removing logarithm, we have
a + b = ab
a – ab = -b
a(1 - b) = -b
a = -b/(1 - b)
Hence,
a = b/(b - 1)

8. Prove that:
(i) $(\log a)^2 - (\log b)^2 = \log (a/b). \log (ab)$
(ii) If a log b + b log a - 1 = 0, then $b^a. a^b = 10$
Solution:

(i) Taking L.H.S. we have,
$= (\log a)^2 - (\log b)^2$
= (log a + log b) (log a - log b) [As $x^2 - y^2 = (x + y)(x - y)$]
= (log ab). (log a/b)
= R.H.S.
- Hence proved

(ii) We have, a log b + b log a - 1 = 0
So,
$\log b^a + \log a^b - 1 = 0$
$\log b^a + \log a^b = 1$
$\log b^a a^b = 1$
On removing logarithm, we get
$b^a a^b = 10$
- Hence proved

9. (i) If log (a + 1) = log (4a - 3) - log 3; find a.
(ii) If 2 log y - log x - 3 = 0, express x in terms of y.
(iii) Prove that: $\log_{10} 125 = 3(1 - \log_{10} 2)$.
Solution:

(i) Given, log (a + 1) = log (4a - 3) - log 3
So,
log (a + 1) = log (4a - 3)/3
On removing logarithm on both sides, we have
a + 1 = (4a - 3)/3
3(a + 1) = 4a – 3

$3a + 3 = 4a – 3$
Hence, $a = 6$

(ii) Given, $2\log y - \log x - 3 = 0$
So,
$\log y^2 – \log x = 3$
$\log y^2/x = 3$
On removing logarithm, we have
$y^2/x = 10^3 = 1000$
Hence, $x = y^2/1000$

(iii) Considering the L.H.S., we have
$\log_{10} 125 = \log_{10} (5 \times 5 \times 5)$
$= \log_{10} 5^3$
$= 3\log_{10} 5$
$= 3\log_{10} 10/2$
$= 3(\log_{10} 10 - \log_{10} 2)$
$= 3(1 - \log_{10} 2)$ [Since, $\log_{10} 10 = 1$]
= R.H.S.
- Hence proved

10. Given $\log x = 2m – n$, $\log y = n – 2m$ and $\log z = 3m – 2n$. Find in terms of m and n, the value of $\log x^2y^3/z^4$.
Solution:

We have, $\log x = 2m – n$, $\log y = n – 2m$ and $\log z = 3m – 2n$
Now, considering
$\log x^2y^3/z^4 = \log x^2y^3 – \log z^4$
$= (\log x^2 + \log y^3) – \log z^4$
$= 2\log x + 3\log y – 4\log z$
$= 2(2m - n) + 3(n – 2m) – 4(3m – 2n)$
$= 4m – 2n + 3n – 6m – 12m + 8n$
$= -14m + 9n$

11. Given $\log_x 25 – \log_x 5 = 2 – \log_x 1/125$; find x.
Solution:

We have, $\log_x 25 – \log_x 5 = 2 – \log_x 1/125$
$\log_x (5 \times 5) – \log_x 5 = 2 – \log_x 1/(5 \times 5 \times 5)$
$\log_x 5^2 – \log_x 5 = 2 – \log_x 1/5^3$
$2\log_x 5 – \log_x 5 = 2 – \log_x 1/5^3$
$\log_x 5 = 2 – 3\log_x 1/5$
$\log_x 5 = 2 + 3\log_x (1/5)^{-1}$
$\log_x 5 = 2 + 3\log_x 5$
$2 = \log_x 5 - 3\log_x 5$
$2 = -2\log_x 5$

$-1 = \log_x 5$
Removing logarithm, we get
$x^{-1} = 5$
Hence, $x = 1/5$

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