NCERT Solutions for Class-XI Maths

Chapter-9 Exercise-9.2

NCERT Math Class 11

1. Find the sum of odd integers from 1 to 2001.

1. The odd integers from 1 to 2001 are 1, 3, $5 \ldots 1999$, 2001.

This sequence forms an A.P.

Here, first term, $a = 1$

Common difference, $d = 2$

Here, $a + (n-1)d = 2001$

$\Rightarrow 1 + (n-1)(2) = 2001$

$\Rightarrow 2n - 2 = 2000$

$\Rightarrow n = 1001$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_n = \frac{1001}{2}[2 \times 1 + (1001-1) \times 2]$

$= \frac{1001}{2}[2 + 1000 \times 2]$

$= \frac{1001}{2} \times 2002$

$= 1001 \times 1001$

$= 1002001$

Thus, the sum of odd numbers from 1 to 2001 is 1002001 .

2. Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

2. The natural numbers lying between 100 and 1000, which are multiples of 5, are 105, 110, … 995.

Let the first term be ‘a’ and common difference ‘d’.

Let n be the total number of terms in the series.

Here, first term, $a = 105$

Common difference, $d = 5$

If l denotes the last term of the series

Then, $l = a + (n - 1) \times d$

Here, $l = 995$

$\Rightarrow 995 = 105 + (n - 1) \times 5$

$\Rightarrow 995 - 105 = (n - 1) \times 5$

$\Rightarrow 890/5 = n - 1$

$\Rightarrow 178 + 1 = n$

$\therefore n = 179$

Sum of A.P. = $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_n = \frac{179}{2}[105 \times 2 + (179 - 1) \times 5]$

$\Rightarrow S_n = \frac{179}{2}[210 + 178 \times 5] = \frac{179}{2} \times [210 + 890] = \frac{179}{2} \times 1100$

$\Rightarrow S_n = 179 \times 550 = 98,450.$

3. In an A.P, the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

3. First term $= 2$

Let d be the common difference of the A.P.

Therefore, the A.P. is $2, 2+d, 2+2d, 2+3d \ldots$

Sum of first five terms $= 10 + 10d$

Sum of next five terms $= 10 + 35d$

According to the given condition,

$10 + 10d = \frac{1}{4}(10 + 35d)$

$\Rightarrow 40 + 40d = 10 + 35d$

$\Rightarrow 30 = -5d$

$\Rightarrow d = -6$

$\therefore a_{20} = a + (20 - 1)d = 2 + (19)(-6) = 2 - 114 = -112$

Thus, the 20^{th} term of the A.P. is -112.

4. How many terms of the A.P. $-6, -\frac{11}{2}, -5, \ldots$ are needed to give the sum -25?

4. First term a = -6

Second term = -11/2

Let d be the common difference.

d = Second term – First term

d = -11/2 – (-6) = 6 – 11/2 = ½

S_n = Sum of n terms of AP = -25

$S_n = \frac{n}{2}[2a + (n-1)d]$

$-25 = \frac{n}{2}\left[2 \times (-6) + (n-1) \times \frac{1}{2}\right]$

$-50 = -12n + \frac{n^2}{2} - \frac{n}{2}$

$-100 = -25n + n^2$

$\Rightarrow n^2 - 25n + 100 = 0$

$\Rightarrow n^2 - 20n - 5n + 100 = 0$

$\Rightarrow n \times (n - 20) - 5 \times (n - 20) = 0$

$\Rightarrow (n - 20) \times (n - 5) = 0$

$\Rightarrow n = 20$ or 5

5. In an A.P., if p^{th} term is $1/q$ and q^{th} term is $1/p$, prove that the sum of first pq terms is $1/2(pq+1)$, where $p \neq q$.

5. It is known that the general term of an A.P. is $a_n = a + (n-1)d$: According to the given information,

p^{th} term $= a_p = a + (p-1)d = \frac{1}{q}$ ….(1)

q^{th} term $= a_q = a + (q-1)d = \frac{1}{p}$ ….(2)

Subtracting (2) from (1), we obtain

$(p-1)d - (q-1)d = \frac{1}{q} - \frac{1}{p}$

$\Rightarrow (p-1-q+1)d = \frac{p-q}{pq}$

$\Rightarrow (p-q)d = \frac{p-q}{pq}$

$\Rightarrow d = \frac{1}{pq}$

Putting the value of d in (1), we obtain

$a + (p-1)\frac{1}{pq} = \frac{1}{q}$

$\Rightarrow a = \frac{1}{q} - \frac{1}{q} + \frac{1}{pq} = \frac{1}{pq}$

$\therefore S_{pq} = \frac{pq}{2}\left[2a + (pq-1)d\right]$

$= \frac{pq}{2}\left[\frac{2}{pq} + (pq-1)\frac{1}{pq}\right]$

$= 1 + \frac{1}{2}(pq-1)$

$= \frac{1}{2}pq + 1 - \frac{1}{2} = \frac{1}{2}pq + \frac{1}{2}$

$= \frac{1}{2}(pq+1)$

Thus, the sum of first pq terms of the A.P. is $\frac{1}{2}(pq+1)$.

6. If the sum of a certain number of terms of the A.P. 25, 22, 19, … is 116. Find the last term.

6. Given terms of A.P. 25, 22, 19, …………..

Let the sum of *n* terms of the given A.P. be 116.

$S_n = 116$

First Term = a = 25, Common Difference = d = 22 – 19 = -3

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Putting the value of a, d and S_n

$\Rightarrow 116 = \frac{n}{2}\left[2 \times 25 + (n-1) \times (-3)\right]$

$\Rightarrow 116 \times 2 = n\,[50 - 3n + 3]$

$\Rightarrow 232 = 53n - 3n^2 \Rightarrow 3n^2 - 53n + 232 = 0$

$\Rightarrow 3n^2 - 24n - 29n + 232 = 0$

$\Rightarrow 3n\,(n-8) - 29\,(n-8) = 0$

$\Rightarrow (3n-29)\,(n-8) = 0$

$\Rightarrow n = 8$ or $29/3$

n cannot be equal to 29/3. Therefore, n = 8

Last term = $a_8 = a + (n-1)d$

$a_8 = 25 + (8-1) \times (-3) = 25 - 21$

$\therefore a_8 = 4$

7. Find the sum to n terms of the A.P., whose k^{th} term is $5k+1$.

7. It is given that the k^{th} term of the A.P. is $5k+1$.

k^{th} term $= a_k = a + (k-1)d$

$\therefore a + (k-1)d = 5k + 1a + kd - d = 5k + 1$

$\therefore$ Comparing the coefficient of k, we obtain $d = 5a - d = 1$

$\Rightarrow a - 5 = 1$

$\Rightarrow a = 6$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{n}{2}\left[2(6) + (n-1)(5)\right]$

$= \frac{n}{2}[12 + 5n - 5]$

$= \frac{n}{2}(5n + 7)$

8. If the sum of n terms of an A.P. is $(pn + qn^2)$, where p and q are constants, find the common difference.

8. Given,

Sum of n terms of A.P. = $pn + qn^2$

We know, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\therefore\ pn + qn^2 = na + \frac{n^2d}{2} - \frac{nd}{2}$

$\Rightarrow pn + qn^2 = \left(a - \frac{d}{2}\right)n + \frac{n^2d}{2}$

Comparing the coefficients of n^2 on both sides, we obtain

$\frac{d}{2} = q$

$\Rightarrow d = 2q$

$\therefore$ the common difference of the A.P. is 2q.

9. The sums of n terms of two arithmetic progressions are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 18^{th} terms.

9. Let a_1, a_2, and d_1, d_2 be the first terms and the common difference of the first and second arithmetic progression respectively.

According to the given condition,

$$\frac{\text{Sum of n terms of first A.P.}}{\text{Sum of n terms of second A.P.}} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]}=\frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}=\frac{5n+4}{9n+6}$$

Substituting $n=35$ in (1), we obtain

$$\frac{2a_1+34d_1}{2a_2+34d_2}=\frac{5(35)+4}{9(35)+6}$$

$$\Rightarrow \frac{a_1+17d_1}{a_2+17d_2}=\frac{179}{321}$$

$$\frac{18^{th}\text{ term of first A.P.}}{18^{th}\text{ term of second A.P}}=\frac{a_1+17d_1}{a_2+17d_2}$$

From (2) and (3), we obtain

$$\frac{18^{th}\text{ term of first A.P.}}{18^{th}\text{ term of second A.P.}}=\frac{179}{321}$$

Thus, the ratio of 18^{th} term of both the A.P.s is $179:321$.

10. If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first (p + q) terms.

10. Let a and d be the first term and the common difference of the A.P. respectively.

Given,

Sum of first p terms = $S_p=\frac{p}{2}\left[2a+(p-1)d\right]$

Sum of first q terms = $S_q=\frac{q}{2}\left[2a+(q-1)d\right]$

$S_p = S_q$

$\frac{p}{2}\left[2a+(p-1)d\right]=\frac{q}{2}\left[2a+(q-1)d\right]$

$\Rightarrow p\,[2a + pd - d] = q\,[2a + qd - d]$

$\Rightarrow 2ap + p\,(p - 1)d = 2aq + q\,(q - 1)d$

$\Rightarrow 2a\,(p - q) + d\,[p(p - 1) - q(q - 1)] = 0$

$\Rightarrow 2a\,(p - q) + d\,[p^2 - q^2 - (p - q)] = 0$

$\Rightarrow 2a\,(p - q) + d\,[(p + q)(p - q) - (p - q)] = 0$

$\Rightarrow (p - q)\,[2a + d\,(p + q - 1)] = 0$

$\Rightarrow [\, 2a + d\,(p + q - 1)] = 0$

$\Rightarrow\ d = -\dfrac{2a}{p+q-1}$

$\therefore\ S_{p+q} = \dfrac{(p+q)}{2}\left[2a + (p+q-1)\times\left(-\dfrac{2a}{(p+q-1)}\right)\right]$

$\Rightarrow\ S_{p+q} = \dfrac{(p+q)}{2}[2a - 2a] = \dfrac{(p+q)}{2}\times 0$

$\Rightarrow S_{p+q} = 0$

11. Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that $\dfrac{a}{p}(q-r) + \dfrac{b}{q}(r-p) + \dfrac{c}{r}(p-q) = 0$

11. Let a_1 and d be the first term and the common difference of the A.P. respectively.

According to the given information,

$S_p = \dfrac{p}{2}\left[2a_1 + (p-1)d\right] = a$

$\Rightarrow 2a_1 + (p-1)d = \dfrac{2a}{p}$ ....(1)

$S_q = \dfrac{q}{2}\left[2a_1 + (q-1)d\right] = b$

$\Rightarrow 2a_1 + (q-1)d = \dfrac{2b}{q}$ ....(2)

$S_r = \dfrac{r}{2}\left[2a_1 + (r-1)d\right] = c$

$\Rightarrow 2a_1 + (r-1)d = \dfrac{2c}{r}$ ....(3)

Subtracting (2) from (1), we obtain

$(p-1)d - (q-1)d = \dfrac{2a}{p} - \dfrac{2b}{q}$

$\Rightarrow d(p-1-q+1) = \dfrac{2aq-2bq}{pq}$

$\Rightarrow d(p-q) = \dfrac{2aq-2bp}{pq}$

$\Rightarrow d = \dfrac{2(aq-bp)}{pq(p-q)}$

Subtracting (3) from (2), we obtain

$$(q-1)d-(r-1)d=\frac{2b}{q}-\frac{2c}{r}$$

$$\Rightarrow d(q-1-r+1)=\frac{2b}{q}-\frac{2c}{r}$$

$$\Rightarrow d(q-r)=\frac{2br-2qc}{qr}$$

$$\Rightarrow d=\frac{2(br-qc)}{qr(q-r)} \qquad \ldots.(4)$$

Equating both the values of d obtained in (4) and (5), we obtain

$$\frac{aq-bp}{pq(p-q)}=\frac{br-qc}{qr(q-r)}$$

$$\Rightarrow qr(q-r)(aq-bq)=pq(p-q)(br-qc)$$

$$\Rightarrow r(aq-bp)(q-r)=p(br-qc)(p-q)$$

$$\Rightarrow (aqr-bpr)(q-r)=(bpr-pqc)(p-q)$$

Dividing both sides by pqr, we obtain

$$\left(\frac{a}{p}-\frac{b}{q}\right)(q-r)=\left(\frac{b}{q}-\frac{c}{r}\right)(p-q)$$

$$\Rightarrow \frac{a}{p}(q-r)-\frac{b}{q}(q-r+p-q)+\frac{c}{r}(p-q)=0$$

$$\Rightarrow \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

Thus, the given result is proved.

12. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of mth and nth term is (2m – 1) : (2n – 1).

12. Let a and d be the first term and common difference of the A.P.

Given,

Sum of m terms of A.P. = S_m

$$S_m=\frac{m}{2}\left[2a+(m-1)d\right]$$

Sum of n terms of A.P. = S_n

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

$$\frac{\text{Sum of m terms}}{\text{Sum of n terms}}=\frac{m^2}{n^2}$$

$$\Rightarrow \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]}=\frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d}=\frac{m}{n}$$

Putting m = 2m – 1 and n = 2n – 1 in the above equation

$$\Rightarrow \frac{2a+(2m-2)d}{2a+(2n-2)d}=\frac{2m-1}{2n-1}$$

$$\Rightarrow \frac{a+(m-1)d}{a+(n-1)d}=\frac{2m-1}{2n-1}$$

$$\frac{\text{mth term of A.P.}}{\text{nth term of A.P.}}=\frac{a+(m-1)d}{a+(n-1)d}$$

$$\Rightarrow \frac{\text{mth term of A.P.}}{\text{nth term of A.P.}}=\frac{2m-1}{2n-1}$$

∴ Ratio of m^{th} and n^{th} term is (2m – 1) : (2n – 1).

13. If the sum of n terms of an A.P. is $2n^2+5n$ and its m^{th} term is 164, find the value of m.

13. Let a and b be the first term and the common difference of the A.P. respectively.

$a_m = a+(m-1)d = 164\ldots$ (1)

Sum of n terms: $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

Here, $\frac{n}{2}\left[2a+nd-d\right]=3n^2+5n$

$\Rightarrow na+n^2\cdot\frac{d}{2}=3n^2+5n$

Comparing the coefficient of n^2 on both sides, we obtain $\frac{d}{2}=3$

$\Rightarrow d=6$

Comparing the coefficient of n on both sides, we obtain

$a-\frac{d}{2}=5$

$\Rightarrow a-3=5$

$\Rightarrow a=8$

Therefore, from (1), we obtain

$8+(m-1)6=164$

$\Rightarrow (m-1)6=164-8=156$

$\Rightarrow m-1=26$

$\Rightarrow m = 27$

Thus, the value of m is 27 .

14. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

14. Let A_1, A_2, A_3, A_4, and A_5 be five numbers between 8 and 26

such that 8, A_1, A_2, A_3, A_4, A_5, 26 is an A.P.

Here, First term = a = 8,

Last Term = b = 26,

Total no. of terms = n = 7

Therefore, 26 = 8 + (7 – 1) *d*

$\Rightarrow 6d = 26 - 8 = 18$

$\Rightarrow d = 3$

$A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 2 \times 3 = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 3 \times 3 = 8 + 9 = 17$

$A_4 = a + 4d = 8 + 4 \times 3 = 8 + 12 = 20$

$A_5 = a + 5d = 8 + 5 \times 3 = 8 + 15 = 23$

Thus, the required five numbers between 8 and 26 are 11, 14, 17, 20, and 23.

15. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then find the value of n .

15. A.M. of a and $b = \frac{a+b}{2}$

According to the given condition,

$$\frac{a+b}{2} = \frac{a^n + b^n}{a^{n-1} + b^{n-1}}$$

$$\Rightarrow (a+b)(a^{n-1} + b^{n-1}) = 2(a^n + b^n)$$

$$\Rightarrow a^n + ab^{n-1} + ba^{n-1} + b^n = 2a^n + 2b^n$$

$$\Rightarrow ab^{n-1} + a^{n-1}b = a^n + b^n$$

$$\Rightarrow ab^{n-1} - b^n = a^n - a^{n-1}b$$

$$\Rightarrow b^{n-1}(a-b) = a^{n-1}(a-b)$$

$$\Rightarrow b^{n-1} = a^{n-1}$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-1} = 1 = \left(\frac{a}{b}\right)^0$$

$$\Rightarrow n - 1 = 0$$

$$\Rightarrow n = 1$$

16. Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of 7^{th} and $(m-1)^{th}$ numbers is 5 : 9. Find the value of m.

16. Let $A_1, A_2, \ldots A_m$ be m numbers such that $1, A_1, A_2, \ldots A_m, 31$ is an A.P.

Here, First term = $a = 1$,

Last term = $b = 31$,

Total no. of terms = $n = m + 2$

$\therefore 31 = 1 + (m + 2 - 1)\, d$

$\Rightarrow 30 = (m + 1)d$

$\Rightarrow \ d = \frac{30}{m+1}$

$A_1 = a + d$

$A_2 = a + 2d$

$A_3 = a + 3d \ldots$

$\therefore A_7 = a + 7d$

$A_{m-1} = a + (m - 1)d$

Also given,

$$\frac{7\text{th term of A.P.}}{(m-1)\text{th term of A.P.}} = \frac{5}{9}$$

$$\Rightarrow \frac{a+7d}{a+(m-1)d} = \frac{5}{9}$$

$$\Rightarrow \frac{1+7\times\frac{30}{m+1}}{1+(m-1)\times\frac{30}{(m+1)}} = \frac{5}{9}$$

$$\Rightarrow \frac{m+1+210}{m+1+30m-30} = \frac{5}{9}$$

$$\Rightarrow \frac{m+211}{31m-29} = \frac{5}{9}$$

$\Rightarrow 9\,(m + 211) = 5\,(31m - 29)$

$\Rightarrow 9m + 1899 = 155m - 145$

$\Rightarrow 155m - 9m = 1899 + 145$

$\Rightarrow 146m = 2044$

$\therefore m = 14$

17. A man starts repaying a loan as first installment of Rs. 100. If he increases the installment by Rs 5 every month, what amount he will pay in the 30^{th} installment?

17. The first installment of the loan is Rs 100.

The second installment of the loan is Rs 105 and so on.
The amount that the man repays every month forms an A.P.
The A.P. is $100, 105, 110 \ldots$
First term, $a = 100$
Common difference, $d = 5$

$A_{30} = a + (30 - 1)d$
$= 100 + (29)(5)$
$= 100 + 145$
$= 245$

Thus, the amount to be paid in the 30^{th} installment is Rs 245 .

18. The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon.

18. Smallest Angle = 102°
Difference between any two consecutive interior angles of a polygon = 5°
The angles of the polygon will form an A.P. with common difference d as 5° and first term a as 120°.
We know, sum of all angles of a polygon with n sides = 180° (n – 2)
$S_n = 180° (n - 2)$

$\text{Also}, S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Equating both we get

$\frac{n}{2}\left[2a + (n-1)d\right] = 180°(n-2)$

$\Rightarrow \frac{n}{2}\left[2 \times 120° + (n-1) \times 5°\right] = 180°(n-2)$

$\Rightarrow n (240 + 5n - 5) = 360n - 720$
$\Rightarrow 5n^2 + 240n - 5n - 360n + 720 = 0$
$\Rightarrow 5n^2 - 125n + 720 = 0$
$\Rightarrow n^2 - 25n + 144 = 0$
$\Rightarrow n^2 - 16n - 9n + 144 = 0$
$\Rightarrow n (n - 16) - 9 (n - 16) = 0$
$\Rightarrow (n - 9) (n - 16) = 0$
$\therefore n = 9 \text{ or } 16$