

EXERCISE 3.5

Construct the composition table for \times_4 on set $S = \{0, 1, 2, 3\}$.

Solution:

Given that \times_4 on set $S = \{0, 1, 2, 3\}$

Here,

$$1 \times_4 1 = \text{remainder obtained by dividing } 1 \times 1 \text{ by } 4 \\ = 1$$

$$0 \times_4 1 = \text{remainder obtained by dividing } 0 \times 1 \text{ by } 4 \\ = 0$$

$$2 \times_4 3 = \text{remainder obtained by dividing } 2 \times 3 \text{ by } 4 \\ = 2$$

$$3 \times_4 3 = \text{remainder obtained by dividing } 3 \times 3 \text{ by } 4 \\ = 1$$

So, the composition table is as follows:

\times_4	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

1. Construct the composition table for $+_5$ on set $S = \{0, 1, 2, 3, 4\}$

Solution:

$$1 +_5 1 = \text{remainder obtained by dividing } 1 + 1 \text{ by } 5 \\ = 2$$

$$3 +_5 1 = \text{remainder obtained by dividing } 3 + 1 \text{ by } 5 \\ = 2$$

$$4 +_5 1 = \text{remainder obtained by dividing } 4 + 1 \text{ by } 5 \\ = 3$$

So, the composition table is as follows:

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

2. Construct the composition table for \times_6 on set $S = \{0, 1, 2, 3, 4, 5\}$.

Solution:

Here,

$1 \times_6 1 =$ remainder obtained by dividing 1×1 by 6

$= 1$

$3 \times_6 4 =$ remainder obtained by dividing 3×4 by 6

$= 0$

$4 \times_6 5 =$ remainder obtained by dividing 4×5 by 6

$= 2$

So, the composition table is as follows:

\times_6	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

3. Construct the composition table for \times_5 on set $Z_5 = \{0, 1, 2, 3, 4\}$

Solution:

Here,

$$1 \times_5 1 = \text{remainder obtained by dividing } 1 \times 1 \text{ by } 5 \\ = 1$$

$$3 \times_5 4 = \text{remainder obtained by dividing } 3 \times 4 \text{ by } 5 \\ = 2$$

$$4 \times_5 4 = \text{remainder obtained by dividing } 4 \times 4 \text{ by } 5 \\ = 1$$

So, the composition table is as follows:

\times_5	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

4. For the binary operation \times_{10} set $S = \{1, 3, 7, 9\}$, find the inverse of 3.

Solution:

Here,

$$1 \times_{10} 1 = \text{remainder obtained by dividing } 1 \times 1 \text{ by } 10 \\ = 1$$

$$3 \times_{10} 7 = \text{remainder obtained by dividing } 3 \times 7 \text{ by } 10 \\ = 1$$

$$7 \times_{10} 9 = \text{remainder obtained by dividing } 7 \times 9 \text{ by } 10 \\ = 3$$

So, the composition table is as follows:

\times_{10}	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

From the table we can observe that elements of first row are same as the top-most row.

So, $1 \in S$ is the identity element with respect to \times_{10}

Now we have to find inverse of 3

$$3 \times_{10} 7 = 1$$

So the inverse of 3 is 7.



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