Exercise

Question 1.

Find the area of a triangle, whose sides are:

(i) 10cm, 24cm and 26cm

Solution:-

Sides of Δ are

a=10cm

b=24cm

c=26cm

$s = \frac{a+b+c}{2} = \frac{10+24+26}{2}$ (Simplifying we get)

$= \frac{60}{2} = 30$

Area of $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$ (Formula)

$= \sqrt{30(30-10)(30-24)(30-26)} = \sqrt{30 \times 20 \times 6 \times 4} = \sqrt{10 \times 3 \times 10 \times 2 \times 2 \times 3 \times 2 \times 2}$

$= \sqrt{10 \times 10 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2}$ Ans $= 10 \times 3 \times 2 \times 2 = 120\text{cm}^2$

(ii) 18mm, 24 mm and 30mm

Solution:-

Sides of Δ are

a=18mm

b=24mm

C=30mm

$S = \frac{a+b+c}{2} = \frac{18+24+30}{2} = \frac{72}{2} = 36$

Area of $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$ (Formula)

$= \sqrt{36(36-18)(36-24)(36-30)} = \sqrt{36 \times 18 \times 12 \times 6} = \sqrt{18 \times 2 \times 18 \times 2 \times 6 \times 6}$

$= \sqrt{18 \times 18 \times 2 \times 2 \times 6 \times 6}$ Ans $= 18 \times 2 \times 6 = 216\text{mm}^2$

(iii) 21 m, 28 m and 35 m

Solution:-

Sides of Δ are

a=21m

b=28m

c=35m

$S = \frac{a+b+c}{2} = \frac{21+28+35}{2}$ (Simplifying we get)

$= \frac{84}{2} = 42$

Area of $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$ (Formula)

$= \sqrt{42(42-21)(42-28)(42-35)} = \sqrt{42 \times 21 \times 14 \times 7} = \sqrt{7 \times 3 \times 2 \times 3 \times 7 \times 2 \times 7 \times 7}$

$= \sqrt{7 \times 7 \times 7 \times 7 \times 3 \times 3 \times 2 \times 2} = 7 \times 7 \times 3 \times 2 \text{ Ans} = 294\text{m}^2$

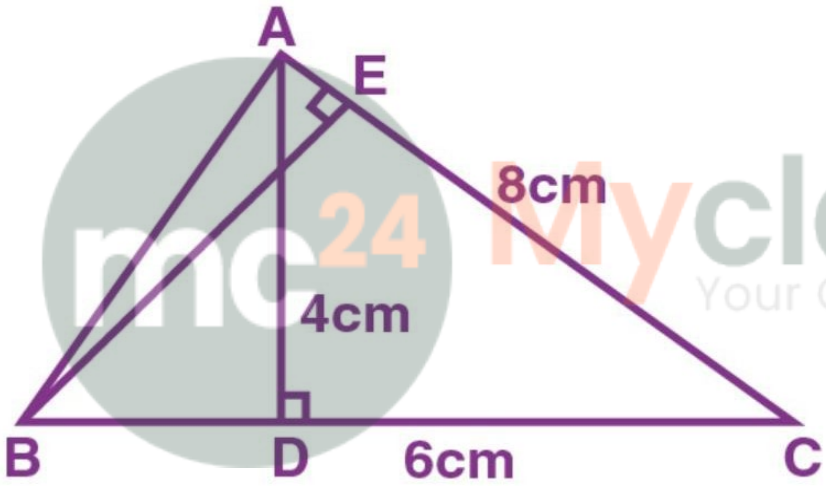
Question 2.

Two sides of a triangle are 6 cm and 8 cm. If height of the triangle corresponding to 6 cm side is 4 cm; find.

(i) Area of the triangle

(ii) Height of the triangle corresponding to 8 cm side.

Solution:-



BC=6cm

Height AD=4cm

Area of $\Delta = \frac{1}{2}$ base $\times$ *height* $= \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 6 \times 4 = 12\text{cm}^2$

Area of $\Delta = \frac{1}{2} AC \times BE\, 12 = \frac{1}{2} \times 8 \times BE \therefore \quad BE = \frac{12 \times 2}{8}$

BE=3cm

(i) 12cm^2 (ii)3cm

Question 3.

The sides of a triangle are 16cm, 12cm and 20cm. Find:

(i) Area of the triangle;

(ii) Height of the triangle, corresponding to the largest side;

(iii) Height of the triangle, corresponding to the smallest side.

Solution:-

Sides of Δ are

a=20cm

b=12cm

c=16cm

$S=\frac{a+b+c}{2}=\frac{20+12+16}{2}=\frac{48}{2}=24$

Area of $\Delta=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{24(24-20)(24-12)(24-16)}=\sqrt{24\times4\times12\times8}$
$=\sqrt{12\times12\times4\times12\times2\times4}=\sqrt{12\times12\times4\times4\times2\times2}=12\times4\times2=96\text{cm}^2$

AD is height of Δ corresponding to largest side.

$\therefore\quad\frac{1}{2}\times BC\times AD=96\ \frac{1}{2}\times20\times\text{AD}=96\ \text{AD}=\frac{96\times2}{20}$

AD=9.6cm

BE is height of Δ corresponding to smallest side.

$\therefore\ \frac{1}{2}AC\times BE=96\ \frac{1}{2}\times12\times BE=96\ BE=\frac{96\times2}{12}$

BE=16cm

(i) 96cm^2 (ii)9.6cm(iii)16cm

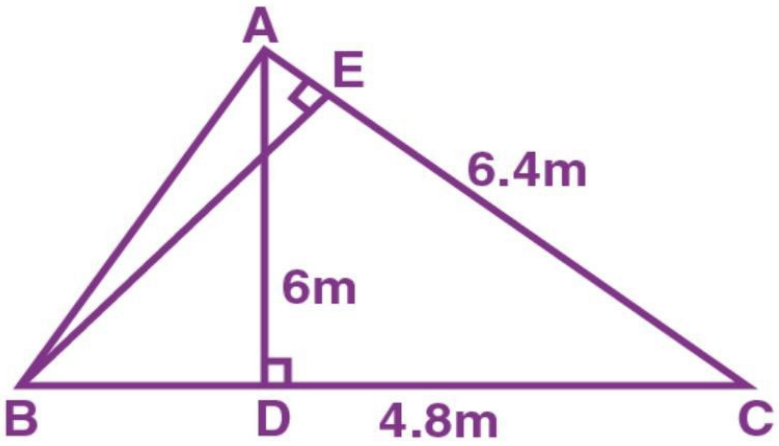
Question 4.

Two sides of a triangle are 6.4 m and 4.8 m. If height of the triangle corresponding to 4.8m side is 6m; find:

(i) Area of the triangle;

(ii) height of the triangle corresponding to 6.4 m side.

Solution:-



ABC is the Δ in which BC=4.8m

AC=6.4m and AD=6m

$\therefore$ Area of $\Delta ABC = \frac{1}{2} BC \times AD = \frac{1}{2} \times 4.8 \times 6 = 14.4 m^2$

BE is height of Δ corresponding to 6.4 m

$\frac{1}{2} AC \times BE = 14.4 \; \frac{1}{2} \times 6.4 \times BE = 14.4 \; BE = \frac{14.4 \times 2}{64} \; BE = \frac{14-4}{3 \cdot 2} = \frac{9}{2} = 4.5 m$

Therefore, (i) $14.4 m^2$ (ii) 4.5 m

Question 5.

The base and the height of a triangle are in the ratio 4:5. If the area of the triangle is 40 m^2; find its base and height.

Solution:-

Consider base of $\Delta = 4 \times m and height of \Delta = 5 \times m$

Area of $\Delta = 40 m^2 \because \frac{1}{2}$ Base $\times$ height $=$ area of $\Delta \; \frac{1}{2} \times 44x \times 5x = 40 \; 10x^2 = 40 \; x^2 = 4$

$x = \sqrt{4}$

x=2

$\therefore$ Base $= 4x = 4 \times 2 = 8m$

Height $= 5x = 5 \times 2 = 10m$

Therefore, the base and height of the triangle is 8m; 10m.

Question 6.

The base and the height of a triangle are in the ratio 5:3. If the area of the triangle is $67 m^2$.find its base and height.

Solution:-

Consider base $= 5 \times m$

Height $= 3 \times m$

Area of $\Delta = \frac{1}{2}$ base $\times height \therefore \quad \frac{1}{2} \times 5x \times 3x = 67.5 \; x^2 = \frac{67.5 \times 2}{15} \; x^2 = 4.5 \times 2 \; x^2 = 9.0$

$x = \sqrt{9}$

x=3

Base $= 5x = 5 \times 3 = 15m$

Height $= 3x = 3 \times 3 = 9m$

Question 7.

The area of an equilateral triangle is $144\sqrt{3}\text{cm}^2$; find its perimeter.

Solution:

Consider each side of an equilateral triangle = x cm

Area $= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}x^2 = 144\sqrt{3} \Rightarrow x^2 = 144\sqrt{3} \times \frac{4}{\sqrt{3}} \Rightarrow x^2 = 144 \times 4 \Rightarrow x^2 = 576$
$\Rightarrow x = \sqrt{576} = 24\text{cm}$

Each side =24cm

Therefore, perimeter =3(24)=72cm

Question 8.

The area of an equilateral triangle is numerically equal to its perimeter. Find its perimeter correct to 2 decimal places.

Solution:-

Consider each side of the equilateral triangle = x

Area $= \frac{\sqrt{3}}{4}x^2$

Area perimeter =3x

By the given condition $= \frac{\sqrt{3}}{4}x^2 = 3x\,x^2 = 3x \times \frac{4}{\sqrt{3}}\,x^1 = \frac{3x \times 4 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{3x \times 4 \times \sqrt{3}}{3} = 4x\sqrt{3}$
$\Rightarrow x^2 = \sqrt{3}(4x) \Rightarrow x = 4\sqrt{3}[\because x \neq 0]$

Perimeter $= 12\sqrt{3}units$

=12(1.732) =20.784=20.78 Units

Question 9.

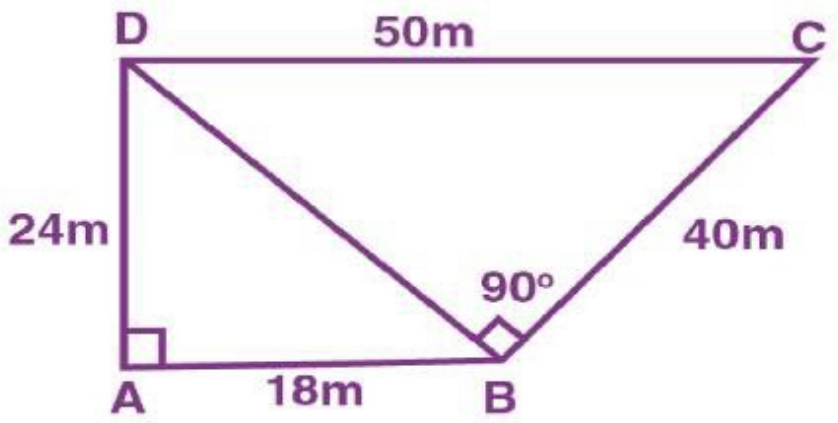
A field is in the shape of a quadrilateral ABCD in which side AB = 18m, side AD = 24m, side BC = 40m, DC = 50m and angle A = 90°. Find the area of the field.

Solution:-

∠A=90°

By Pythagoras Theorem,

In ΔABD,



$\mathrm{BD} = \sqrt{\mathrm{AB}^2 + \mathrm{AD}^2} = \sqrt{18^2 + 24^2} = \sqrt{324 + 576} = \sqrt{900} = 30\mathrm{m}$

Area of $\Delta\mathrm{ABC} = \frac{1}{2}(18)(24) = (18)(12) = 216\mathrm{m}^2$

In ΔBCD; sides are 30, 40, 50

By Pythagoras Theorem $\angle CBD = 90^{\circ}$ $\left[\because, \mathrm{DC}^2 = \mathrm{BD}^2 + \mathrm{BC}^2, \text{ since } (50)^2 = (30)^2 + (40)^2\right]$

Area of $\Delta\mathrm{BCD} = \frac{1}{2}(40)(30) = 600\mathrm{m}^2$

Therefore, area of quadrilateral ABCD= Area of ΔABD+ area of ΔBCD

=216+600=816 m^2

Question 10.

The lengths of the sides of a triangle are in the ratio 4 : 5 : 3 and its perimeter is 96 cm . Find its area.

Solution:-

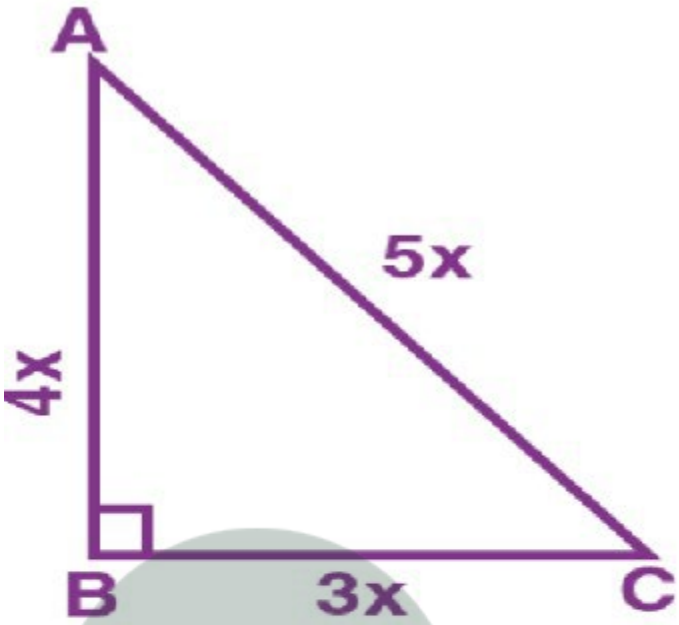
Consider the sides of the triangle ABC be 4 x, 5 x and 3x

AB=4 x, AC=5 x and BC=3 x

Perimeter = 4 x+5x+3x=96

12 x=96

$x = \frac{96}{12}$



x=8

Sides are

BC =3(8) =24 cm AB=4(8) =32 cm,

AC = 5(8) = 40 cm

$(AC)^2 = (AB)^2 + (BC)^2 \left[(5x)^2 = (3x)^2 + (4x)^2\right]$

By Pythagoras Theorem, ∠B=90°

$\text{Area of } \Delta \text{ ABC} = \frac{1}{2}(\text{BC})(\text{AB}) = \frac{1}{2}(24)(32) = 12 \times 32 = 384\text{cm}^2$

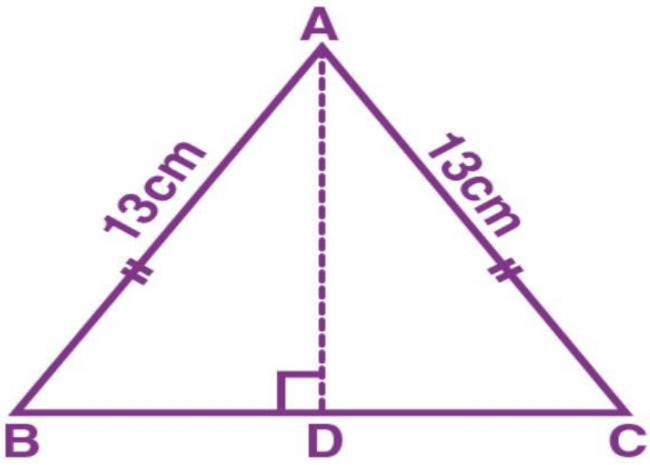
Question 11.

One of the equal sides of an isosceles triangle is 13cm and its perimeter is 50cm. Find the area of the triangle.

Solution:-

In Isosceles ΔABC

AB=AC=13cm But perimeter =50cm



BC=50-(13+13)cm

=50-26=24cm

AD⊥BC

$\mathrm{AD} = \mathrm{DC} = \frac{24}{2} = 12\mathrm{cm}$

In right ΔABD,

$\mathrm{AB}^2 = \mathrm{AD}^2 + \mathrm{BD}^2$ (Pythagoras Theorem)

$(13)^2 = \mathrm{AD}^2 + (12)^2 \Rightarrow 169 = AD^2 + 144 \Rightarrow AD^2 = 169 - 144$

=25=(5)^{2}

AD=5cm.

Area of $\Delta ABC = \frac{1}{2}$ Base $\times$ *Altitude* $= \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 24 \times 5 = 60\mathrm{cm}^2$

Question 12.

The altitude and the base of a triangular field are in the ratio 6:5. If its cost is Rs.49, 57,200 at the rate of Rs.36, 720 per hectare and 1 hectare =10,000 sq. m, find (in meter) dimensions of the field,

Solution:-

Total cost =349,57,200

Rate = 736,720 per hectare

Total area of the triangular field

$= \frac{4957200}{36720} \times 10000\text{m}^2 = 1350000\text{m}^2$

Ratio in altitude and base of the field =6:5

Consider altitude =6x and base =5x

$\text{Area} = \frac{1}{2}\text{ Base} \times \textit{Altitude} \Rightarrow 1350000 = \frac{1}{2} \times 5\text{x} \times 6\text{x} \Rightarrow 15\text{x}^2 = 1350000 \Rightarrow \text{x}^2 = \frac{1350000}{15}$

$\Rightarrow x^2 = 90000 = (300)^2$

x=300

$\text{Base} = 5\text{x} = 5 \times 300 = 1500\text{m}$

$\text{Altitude} = 6\text{x} = 6 \times 300 = 1800\text{m}$

Question 13.

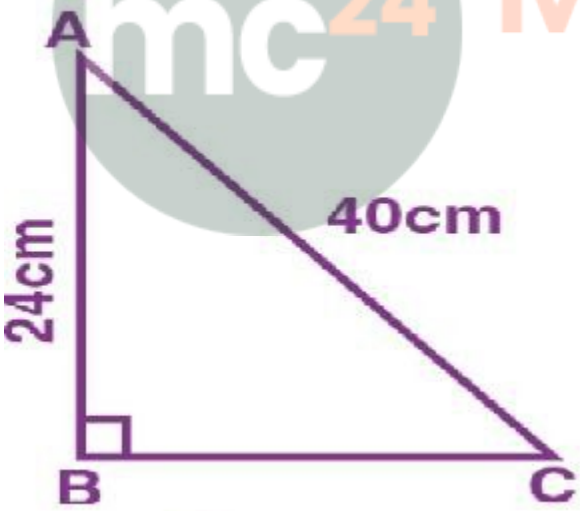
Find the area of the right-angled triangle with hypotenuse 40cm and one of the other two sides 24cm.

Solution:-

In right angled triangle ABC Hypotenuse AC =40cm

One side AB=24cm

$\text{BC} = \sqrt{\text{AC}^2 - \text{AB}^2} = \sqrt{40^2 - 24^2} = \sqrt{1600 - 576} = \sqrt{1024} = 32\text{cm}$



$\text{Area} = \frac{1}{2}\text{AB} \times \text{BC} = \frac{1}{2} \times 24 \times 32\,\text{cm}^2 = 384\text{cm}^2$

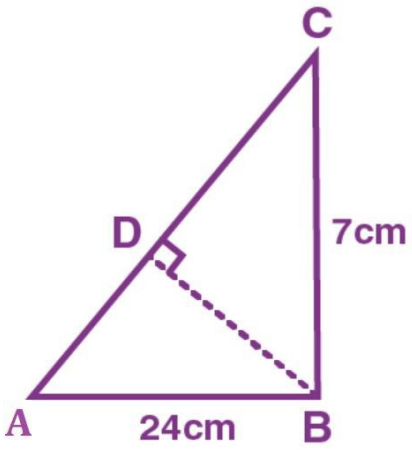
Question 14.

Use the information given in the adjoining figure to find:

(i) The length of AC.

(ii) The area of a ΔABC

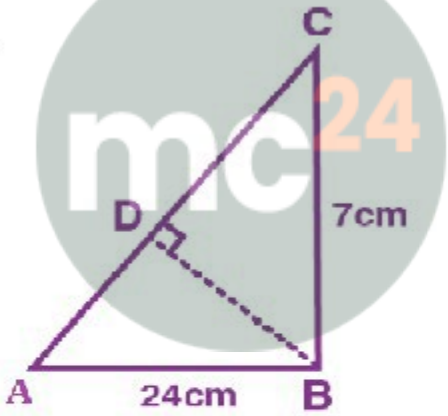
(iii) The length of BD, correct to one decimal place.



Solution:-

Sol. AB=24cm, BC=7cm

(i) $AC = \sqrt{AB^2 + BC^2}$



$= \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25\text{cm}$

(ii) Area of $\triangle ABC = \frac{1}{2} AB \times BC = \frac{1}{2} \times 24 \times 7 = 84\text{cm}^2$

(iii) BD⊥AC

Area $\Delta ABC = \frac{1}{2} AC \times BD \; 84 = \frac{1}{2} \times 25 \times BD \Rightarrow BD = \frac{84 \times 2}{25} = \frac{168}{25} = 6.72\text{cm}$

=6.7cm

Question 15.

Find the length and perimeter of a rectangle, whose area $=120\text{cm}^2$ and breadth =8cm

Solution:-

Area of rectangle $=120\text{cm}^2$
Breadth, b=8cm

Area $= l \times b\, l \times 8 = 120\, l = \frac{120}{8} = 15\text{cm}$

Perimeter $= 2(1 + b) = 2(15 + 8) = 2 \times 23 = 46\text{cm}$

Length =15cm

Perimeter =46cm