Chapter 5: Arithmetic Progressions

EXERCISE 5.4 (Optional)*:

1. Which term of the AP: 121, 117, 113, . . ., isits first negative term?
[Hint: Find *n* for $a_n < 0$]

Solution:Here; a = 121 and d = -4

The first negative term will be less than zero. Moreover, each term gives 1 as remainder when divided by 4. So the last positive term should be5.

Let us check the value of n for an = 5.

5 = a + (n – 1)d

121 + (n – 1) (- 4) = 5

121 – 4n + 4 = 5

125 – 4n = 5

4n = 120

n = 30

Thus, a_{31} = 5 – 4

= 1

Therefore, 32nd term will be the first negative term.

2. The sum of the third and the seventh termsof an AP is 6 and their product is 8. Findthe sum of first sixteen terms of the AP.

Solution:a_3 = a + 2d

a_7 = a + 6d

As per question;

a_3 + a_8 = 6

$a + 2d + a + 6d = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 – 4d$ (i)

Similarly,

$(a + 2d)(a + 6d) = 8$

$a^2 + 6ad + 2ad + 12d^2 = 8$

Substituting the value of a in equation (i), we get;

$(3 – 4d)^2 + 8(3 – 4d)d + 12d^2 = 8$

$9 – 24d + 16d^2 + 24d – 32d^2 + 12d^2 = 8$

$9 – 4d^2 = 8$

$2d = 1$

$d = ½$

Using the value of d in equation (1), we get;

$a = 3 – 4d$

Or, $a = 3 – 2 = 1$

Sum of first 16 terms is calculated as follows:

$$S = \frac{N}{2}[2a + (n - 1)d]$$
$$= \frac{16}{2}\left[2 * 1 + \frac{(15)1}{2}\right]$$

$= 8[2 + (15/2)]$

$=4*19$

$= 76$

3. A ladder has rungs 25 cm apart.(see Fig. 5.7). The rungs decreaseuniformly in length from 45 cm at thebottom to 25 cm at the top. If the top andthe bottom rungs are $2\frac{1}{2}$ m apart, what isthe length of the wood required for therungs?

[Hint : Number of rungs $= \frac{250}{25}$]

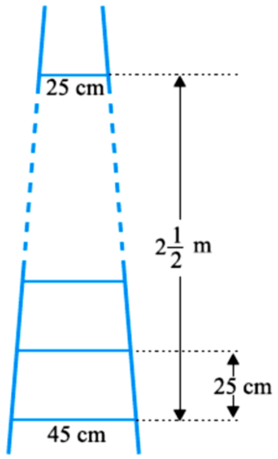

Fig. 5.7

Solution: Number of rungs can be calculated as follows:

2.5 x 100 cm ÷ 25 + 1

= 2.5 x 4 + 1

= 10 + 1 = 11

We have; a = 25, a_{11} = 45 and n = 11

d can be calculated as follows:

$a_{11} = a + 10d$

45 = 25 + 10d

10d = 45 – 25 = 20

d = 2

Total length of wood will be equal to the sum of 11 terms:

$$S = \frac{N}{2}[2a + (n-1)d]$$
$$= \frac{11}{2}[2 * 25 + 10 * 2]$$

= 8[25 + (10)]

= 11 *35

= 385 cm

4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a valueof x such that the sum of the numbers of the houses preceding the house numbered x isequal to the sum of the numbers of the houses following it. Find this value of x.

[Hint : $S_{x-1} = S_{49} - S_x$]

Solution:In this question: a = 1, d = 1 and a_{49} = 49

According to question:

$S_{(x-1)} = S_{49} - S_x$ (i)

The sum of x terms can be calculated as:

$$S_x = \frac{x}{2}[2 + (x-1)1]$$

$$= \frac{x}{2}[x+1]$$

$= (x^2+x)/2$

Now, sum of 49 :

$$S_x = \frac{49}{2}[2 + (48)1]$$

$$= \frac{49}{2}(50)$$

= 1225

Now,

After using the values of S_{x-1}, S_{49} and S_x in (i), we get;

$S_{x-1} = S_{49} - S_x$

$(x^2 - x)/2 = 1225 - (x^2 - x)/2$

$(x^2 - x + x^2 + x)/2$

$(2x^2)/2$

$x^2 = 1225$

x = 35

5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long andbuilt of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. (see Fig. 5.8). Calculate the total volumeof concrete required to build the terrace.

[Hint : Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50 \text{ m}^3$]

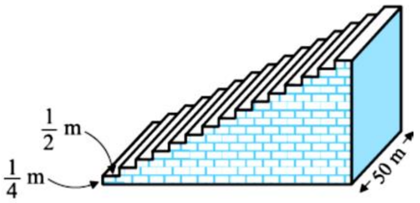

Fig. 5.8

Solution:Dimensions of 1st step = 50 m x 0.25 m x 0.5 m

Volume of 1st step = 6.25 m^3

Dimensions of 2nd step = 50 m x 0.5 m x 0.5 m

Volume of 2ndstep = 12.5 m^3

Dimensions of 3rdstep = 50 m x 0.75 m x 0.5 m

Volume of 3rdstep = 18.75 m^3

Now, we have a = 6.25, d = 6.25 and n = 15

Sum of 15 terms can be calculated as follows:

$$S_{15} = \frac{15}{2}[2 * 6.25 + (14)6.25]$$

$$= \frac{15}{2}(100)$$

= 750

Hence, the volume of concrete will be 750m^3