# Activity 11

### OBJECTIVE

To verify section formula by graphical method.

### MATERIAL REQUIRED

Cardboard, chart paper, graph paper, glue, geometry box and pen/pencil.

### METHOD OF CONSTRUCTION

1. Paste a chart paper on a cardboard of a convenient size.
2. Paste a graph paper on the chart paper.
3. Draw the axes X′OX and Y′OY on the graph paper [see Fig. 1].

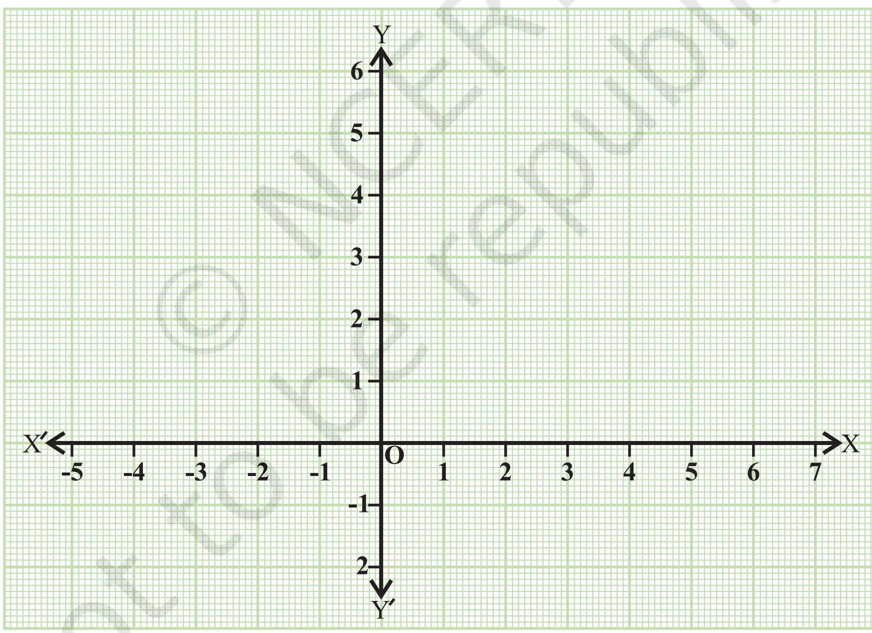


**Fig. 1**

4. Take two points A $(x_1, y_1)$ and B $(x_2, y_2)$ on the graph paper [see Fig. 2].
5. Join A to B to get the line segment AB.

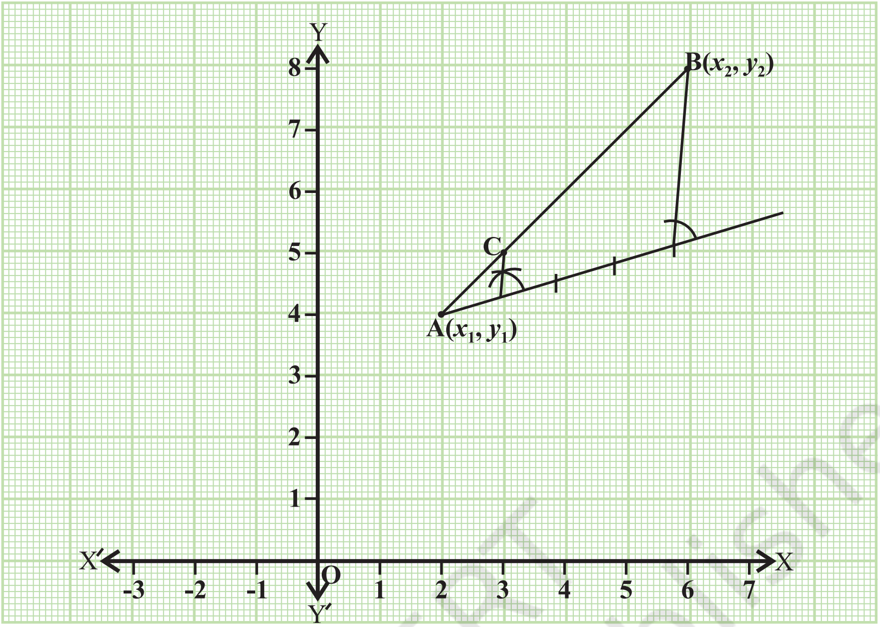


**Fig. 2**

## Demonstration

1. Divide the line segment AB internally (in the ratio $m:n$) at the point C [see Fig. 2].
2. Read the coordinates of the point C from the graph paper.
3. Using section formula, find the coordinates of C.
4. Coordinates of C obtained from Step 2 and Step 3 are the same.

## Observation

1. Coordinates of A are __________.
2. Coordinates of B are __________.
3. Point C divides AB in the ratio __________.
4. Coordinates of C from the graph paper are __________.
5. Coordinates of C by using section formula are __________.
6. Coordinates of C from the graph paper and from section formula are ______.

## Application

This formula is used to find the centroid of a triangle in geometry, vector algebra and 3–dimensional geometry.

# Activity 12

### Objective

To verify the formula for the area of a triangle by graphical method.

### Material Required

Cardboard, chart paper, graph paper, glue, pen/pencil and ruler.

### Method of Construction

1. Take a cardboard of convenient size and paste a chart paper on it [see Fig. 1].
2. Paste a graph paper on the chart paper.
3. Draw the axes X′OX and Y′OY on the graph paper [see Fig. 1].

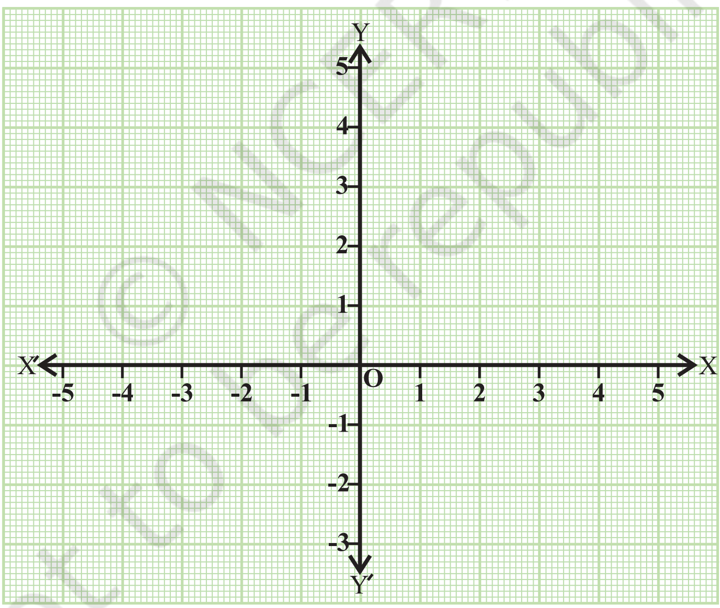


**Fig. 1**

4. Take three points A($x_1$,$y_1$), B($x_2$,$y_2$) and C($x_3$,$y_3$) on the graph paper.
5. Join the points to get a triangle ABC [see Fig. 2].

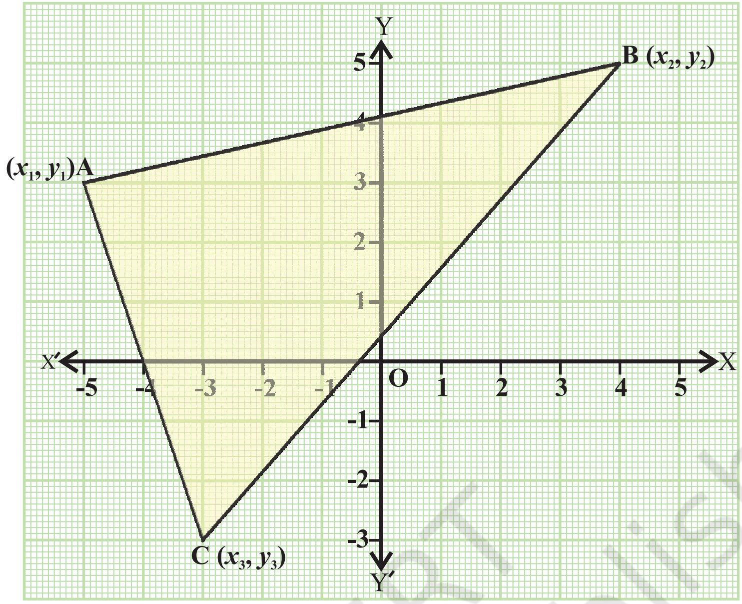


**Fig. 2**

## DEMONSTRATION

1. Calculate the area of the triangle ABC using the formula:

$$\text{Area} = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right].$$

2. Find the area of the triangle ABC by counting the number of squares enclosed in it in the following way:

   (i) take a complete square as 1

   (ii) take more than half a square as 1

   (iii) take half a square as $\frac{1}{2}$

   (iv) ignore the squares which are less than half a square.

3. Area calculated from formula and by actually counting the squares is nearly the same [see steps 1 and 2].

## OBSERVATION

1. Coordinates of A are _______________.

   Coordinates of B are _______________.

   Coordinates of C are _______________.

2. Area of ΔABC by using formula = ______________.

3. (i) Number of complete squares = ______________.

   (ii) Number of more than half squares = ___________.

   (iii) Number of half squares = ______________.

   (iv) Total area by counting the squares = ______________.

4. Area calculated by using formula and by counting the number of squares are ______________.

## APPLICATION

The formula for the area of the triangle is useful in various results in geometry such as checking collinearity of three points, calculating area of a triangle/quadrilateral/polygon.

# Activity 13

## Objective

To establish the criteria for similarity of two triangles.

## Material Required

Coloured papers, glue, sketch pen, cutter, geometry box .

## Method of Construction

**I**

1. Take a coloured paper/chart paper. Cut out two triangles ABC and PQR with their corresponding angles equal.

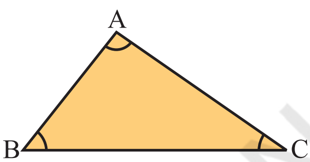


**Fig. 1**

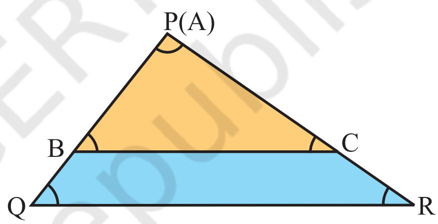


**Fig. 2**

2. In the triangles ABC and PQR, $\angle A = \angle P$; $\angle B = \angle Q$ and $\angle C = \angle R$.
3. Place the $\Delta ABC$ on $\Delta PQR$ such that vertex A falls on vertex P and side AB falls along side PQ (side AC falls along side PR) as shown in Fig. 2.

## Demonstration I

1. In Fig. 2, $\angle B = \angle Q$. Since corresponding angles are equal, BC||QR
2. By BPT, $\dfrac{PB}{BQ} = \dfrac{PC}{CR}$ or $\dfrac{AB}{BQ} = \dfrac{AC}{CR}$

or $\dfrac{BQ}{AB} = \dfrac{CR}{AC}$

or $$\frac{BQ + AB}{AB} = \frac{CR + AC}{AC}$$ [Adding 1 to both sides]

or $$\frac{AQ}{AB} = \frac{AR}{AC} \text{ or } \frac{PQ}{AB} = \frac{PR}{AC} \text{ or } \frac{AB}{PQ} = \frac{AC}{PR} \qquad (1)$$

**II**

1. Place the $\Delta ABC$ on $\Delta PQR$ such that vertex B falls on vertex Q, and side BA falls along side QP (side BC falls along side QR) as shown in Fig. 3.

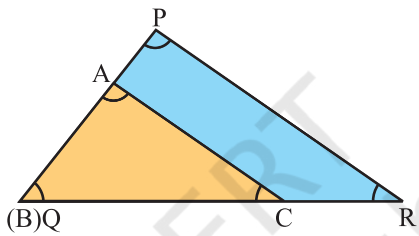


**Fig. 3**

## Demonstration II

1. In Fig. 3, $\angle C = \angle R$. Since corresponding angles are equal, $AC \parallel PR$

2. By BPT, $\frac{AP}{AB} = \frac{CR}{BC}$; or $\frac{BP}{AB} = \frac{BR}{BC}$ [Adding 1 on both sides]

or $$\frac{PQ}{AB} = \frac{QR}{BC} \text{ or } \frac{AB}{PQ} = \frac{BC}{QR} \qquad (2)$$

From (1) and (2), $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

Thus, from Demonstrations I and II, we find that when the corresponding angles of two triangles are equal, then their corresponding sides are proportional. Hence, the two triangles are similar. This is AAA criterion for similarity of triangles.

**Alternatively,** you could have measured the sides of the triangles ABC and PQR and obtained

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}.$$

From this result, ΔABC and ΔPQR are similar, i.e., if three corresponding angles are equal, the corresponding sides are proportional and hence the triangles are similar. This gives AAA criterion for similarity of two triangles.

### III

1. Take a coloured paper/chart paper, cut out two triangles ABC and PQR with their corresponding sides proportional.

i.e., $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

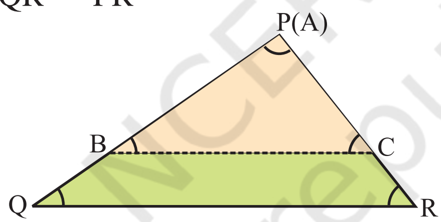


**Fig. 4**

2. Place the ΔABC on ΔPQR such that vertex A falls on vertex P and side AB falls along side PQ. Observe that side AC falls along side PR [see Fig. 4].

## DEMONSTRATION III

1. In Fig. 4, $\frac{AB}{PQ} = \frac{AC}{PR}$. This gives $\frac{AB}{BQ} = \frac{AC}{CR}$. So, BC||QR (by converse of BPT)

i.e., ∠B = ∠Q and ∠C = ∠R. Also ∠A = ∠P. That is, the corresponding angles of the two triangles are equal.

Thus, when the corresponding sides of two triangles are proportional, their corresponding angles are equal. Hence, the two triangles are similar. This is the SSS criterion for similarity of two triangles.

**Alternatively**, you could have measured the angles of $\Delta ABC$ and $\Delta PQR$ and obtained $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$.

From this result, $\Delta ABC$ and $\Delta PQR$ are similar, i.e., if three corresponding sides of two triangles are proportional, the corresponding angles are equal, and hence the triangles are similar. This gives SSS criterion for similarity of two triangles.

**IV**

1. Take a coloured paper/chart paper, cut out two triangles ABC and PQR such that their one pair of sides is proportional and the angles included between the pair of sides are equal.

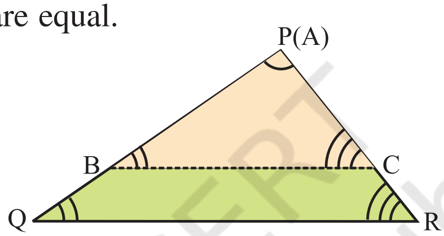


**Fig. 5**

i.e., In $\Delta ABC$ and $\Delta PQR$, $\frac{AB}{PQ} = \frac{AC}{PR}$ and $\angle A = \angle P$.

2. Place triangle ABC on triangle PQR such that vertex A falls on vertex P and side AB falls along side PQ as shown in Fig. 5.

## DEMONSTRATION IV

1. In Fig. 5, $\frac{AB}{PQ} = \frac{AC}{PR}$. This gives $\frac{AB}{BQ} = \frac{AC}{CR}$. So, BC||QR (by converse of BPT)

   Therefore, $\angle B = \angle Q$ and $\angle C = \angle R$.

   From this demonstration, we find that when two sides of one triangle are proportional to two sides of another triangle and the angles included between the two pairs of sides are equal, then corresponding angles of two triangles are equal.

Hence, the two triangles are similar. This is the SAS criterion for similarity of two triangles.

**Alternatively**, you could have measured the remaining sides and angles of $\Delta ABC$ and $\Delta PQR$ and obtained $\angle B = \angle Q$, $\angle C = \angle R$ and

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}.$$

From this, $\Delta ABC$ and $\Delta PQR$ are similar and hence we obtain SAS criterion for similarity of two triangles.

## Observation

By actual measurement:

I. In $\Delta ABC$ and $\Delta PQR$,

$\angle A$ = ______, $\angle P$ = ______, $\angle B$ = ______, $\angle Q$ = ______, $\angle C$ = ______, $\angle R$ = ______,

$\frac{AB}{PQ}$ = ______; $\frac{BC}{QR}$ = ________; $\frac{AC}{PR}$ = ________

If corresponding angles of two triangles are ________, the sides are ________. Hence the triangles are ________.

II. In $\Delta ABC$ and $\Delta PQR$

$\frac{AB}{PQ}$ = ______; $\frac{BC}{QR}$ = ________; $\frac{AC}{PR}$ = ________

$\angle A$ = _______, $\angle B$ = _______, $\angle C$ = _______, $\angle P$ = _______,

$\angle Q$ = _______, $\angle R$ = _______.

If the corresponding sides of two triangles are ______, then their corresponding angles are ________. Hence, the triangles are ________.

III. In $\Delta ABC$ and $\Delta PQR$ ,

$$\frac{AB}{PQ} = \_\_\_\_\_\_\_\_; \quad \frac{AC}{PR} = \_\_\_\_\_\_\_\_\_\_$$

$\angle A$ = ________, $\angle P$ = ________, $\angle B$ = ________, $\angle Q$ = ________,

$\angle C$ = ________, $\angle R$ = ________.

If two sides of one triangle are ______ to the two sides of other triangle and angles included between them are ________, then the triangles are ________.

## Application

The concept of similarity is useful in reducing or enlarging images or pictures of objects.

# Activity 14

### OBJECTIVE

To draw a system of similar squares, using two intersecting strips with nails.

### MATERIAL REQUIRED

Two wooden strips (each of size 1 cm wide and 30 cm long), adhesive, hammer, nails.

### METHOD OF CONSTRUCTION

1. Take two wooden strips say AB and CD.
2. Join both the strips intersecting each other at right angles at the point O [see Fig. 1].
3. Fix five nails at equal distances on each of the strips (on both sides of O) and name them, say $A_1$, $A_2$, ......., $A_5$, $B_1$, $B_2$, ......., $B_5$, $C_1$, $C_2$, ......., $C_5$ and $D_1$, $D_2$, ......., $D_5$ [see Fig. 2].

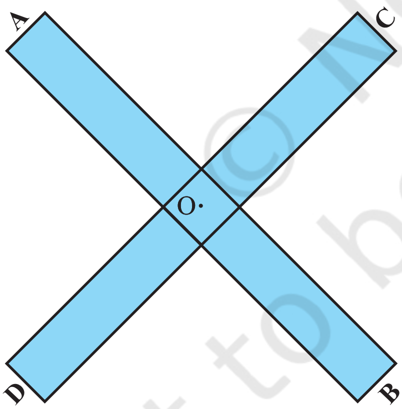

Fig. 1

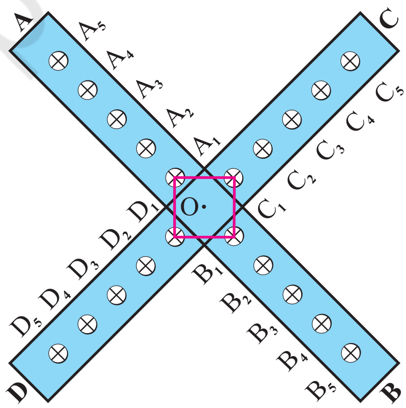

Fig. 2

4. Wind the thread around nails of subscript 1 ($A_1C_1B_1D_1$) on four ends of two strips to get a square [see Fig. 3].
5. Similarly, wind the thread around nails of same subscript on respective strips [see Fig. 3]. We get squares $A_1C_1B_1D_1$ , $A_2C_2B_2D_2$, $A_3C_3B_3D_3$, $A_4C_4$ $B_4$ $D_4$ and $A_5C_5B_5D_5$.

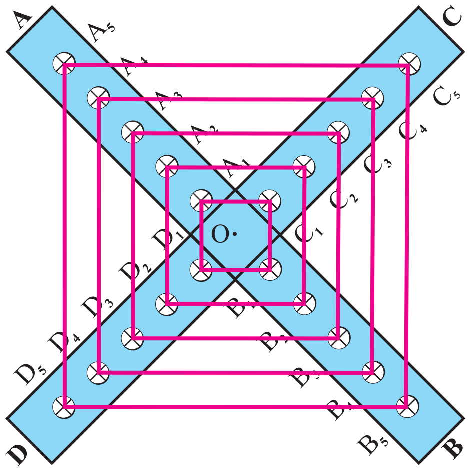


**Fig. 3**

## Demonstration

1. On each of the strips AB and CD, nails are positioned equidistant to each other, such that

   $A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$, $\quad B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$

   $C_1C_2 = C_2C_3 = C_3C_4 = C_4C_5$, $\quad D_1D_2 = D_2D_3 = D_3D_4 = D_4D_5$

2. Now in any one quadrilateral say $A_4C_4B_4D_4$ [see Fig. 3].

   $A_4O = OB_4 = 4$ units.

   Also, $D_4O = OC_4 = 4$ units,

   where 1 unit = distance between two consecutive nails.

   Therefore, diagonals bisect each other.

   Therefore, $A_4C_4B_4D_4$ is a parallelogram.

   Moreover, $A_4B_4 = C_4D_4 = 4 \times 2 = 8$ units, i.e., diagonals are equal to each other.

   In addition to this, $A_4B_4$ is perpendicular to $C_4D_4$ (The strips are perpendicular to each other).

   Therefore, $A_4C_4B_4D_4$ is a square.

Similarly, we can say that $A_1C_1B_1D_1$, $A_2C_2B_2D_2$, $A_3C_3B_3D_3$ and $A_5C_5B_5D_5$ are all squares.

3. Now to show similarity of squares [see Fig. 3], measure $A_1C_1$, $A_2C_2$, $A_3C_3$, $A_4C_4$, $A_5C_5$, $C_1B_1$, $C_2B_2$, $C_3B_3$, $C_4B_4$, $C_5B_5$ and so on.

   Also, find ratios of their corresponding sides such as $\frac{A_2C_2}{A_3C_3}, \frac{C_2B_2}{C_3B_3}, \ldots$

## Observation

By actual measurement:

$A_2C_2$ = _______, $A_4C_4$ = _______

$C_2B_2$ = _______, $C_4B_4$ = _______

$B_2D_2$ = _______, $B_4D_4$ = _______

$D_2A_2$ = _______, $D_4A_4$ = _______.

$\frac{A_2C_2}{A_4C_4}$ = _______, $\frac{C_2B_2}{C_4B_4}$ = _______, $\frac{B_2D_2}{B_4D_4}$ = _______, $\frac{D_2A_2}{D_4A_4}$ = _______.

Also, $\angle A_2$ = _______, $\angle C_2$ = _______, $\angle B_2$ = _______,

$\angle D_2$ = _______, $\angle A_4$ = _______, $\angle C_4$ = _______,

$\angle B_4$ = _______, $\angle D_4$ = _______.

Therefore, square $A_2C_2B_2D_2$ and square $A_4C_4B_4D_4$ are _________.

Similarly, each square is _______ to the other squares.

### Note

By taking the lengths of the two diagonals unequal and angle between the strips other than a right angle, we can obtain similar parallelograms/rectangles by adopting the same procedure.

## Application

Concept of similarity can be used in enlargement or reduction of images like maps in atlas and also in making photographs of different sizes from the same negative.

### Precautions

1. Care should be taken while using nails and hammer.
2. Nails should be fixed at equal distances.

# Activity 15

### OBJECTIVE

To draw a system of similar triangles, using Y shaped strips with nails.

### MATERIAL REQUIRED

Three wooden strips of equal lengths (approx. 10 cm long and 1 cm wide), adhesive, nails, cello-tape, hammer.

### METHOD OF CONSTRUCTION

1. Take three wooden strips say P, Q, R and cut one end of each strip [see Fig. 1]. Using adhesive/tape, join three ends of each strip such that they all lie in different directions [see Fig. 1].

2. Fix five nails at equal distances on each of the strips and name them $P_1, P_2, ..., P_5, Q_1, Q_2, ..., Q_5$ and $R_1, R_2, ..., R_5$ on strips P, Q and R, respectively [see Fig. 2].

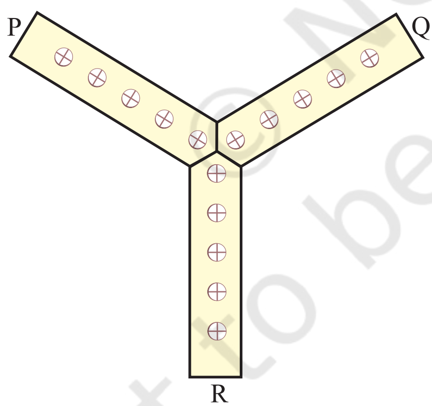


**Fig. 1**

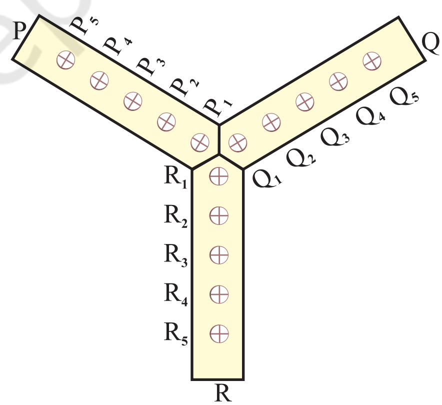


**Fig. 2**

3. Wind the thread around the nails of subscript 1 ($P_1$, $Q_1$, $R_1$) on three respective strips [see Fig. 3].

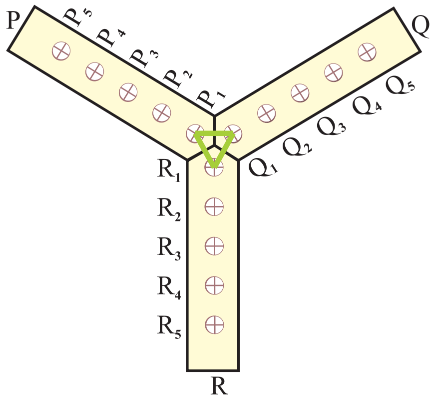


**Fig. 3**

4. To get more triangles, wind the thread around the nails of the same subscript on the respective strips. We get triangles $P_1Q_1R_1$, $P_2Q_2R_2$, $P_3Q_3R_3$, $P_4Q_4R_4$ and $P_5Q_5R_5$ [see Fig. 4].

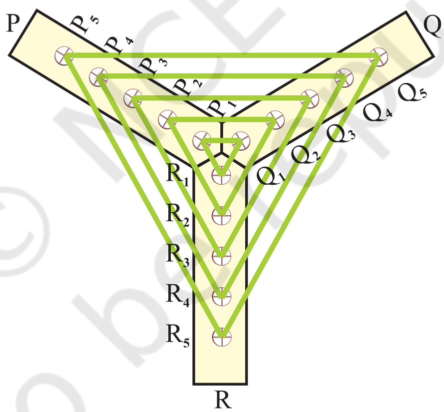


**Fig. 4**

## DEMONSTRATION

1. Three wooden strips are fixed at some particular angles.
2. On each of the strips P, Q, R, nails are positioned at equal distances such that $P_1P_2=P_2P_3=P_3P_4=P_4P_5$ on strip P and similarly $Q_1Q_2=Q_2Q_3=Q_3Q_4=Q_4Q_5$ and $R_1R_2=R_2R_3=R_3R_4=R_4R_5$ on strip Q and R, respectively.

3. Now take any two triangles say $P_1Q_1R_1$ and $P_5Q_5R_5$. Measure the sides $P_1Q_1$, $P_5Q_5$, $P_1R_1$, $P_5R_5$, $R_1Q_1$ and $R_5Q_5$.

4. Find the ratios $\dfrac{P_1Q_1}{P_5Q_5}$, $\dfrac{P_1R_1}{P_5R_5}$ and $\dfrac{R_1Q_1}{R_5Q_5}$.

5. Observe that $\dfrac{P_1Q_1}{P_5Q_5} = \dfrac{P_1R_1}{P_5R_5} = \dfrac{R_1Q_1}{R_5Q_5}$

   Thus, $\Delta P_1Q_1R \sim P_5Q_5R_5$ (SSS similarity criterion)

6. It can be easily shown that any two triangles formed on Y shaped strips are similar.

## Observation

By actual measurement:

$P_1Q_1$ = ______, $Q_1R_1$ = ______, $R_1P_1$ = ______,

$P_5Q_5$ = ______, $Q_5R_5$ = ______, $R_5P_5$ = ______,

$\dfrac{P_1Q_1}{P_5Q_5}$ = ______, $\dfrac{Q_1R_1}{Q_5R_5}$ = ______, $\dfrac{R_1P_1}{R_5P_5}$ = ______

Therefore, $\Delta P_1Q_1R_1$ and $\Delta P_5Q_5R_5$ are ______.

## Application

1. Concept of similarity can be used in reducing or enlarging images (like maps in atlas) and also in making photographs of different sizes from the same negative.

2. Using the concept of similarity, unknown dimensions of an object can be determined using a similar object with known dimensions.

3. Using concept of similarity and length of the shadow of pole in sunlight, height of a pole can be found out.

**Note**

By winding a thread at suitable nails, we can obtain non-equilateral similar triangles also.