EXERCISE 3.4

Graphically, solve the following pair of equations: $2x + y = 6$
$2x - y + 2 = 0$
Find the ratio of the areas of the two triangles formed by the lines representing these equations with the *x*-axis and the lines with the *y*-axis.

Solution:

Given equations are $2x + y = 6$ and $2x - y + 2 = 0$

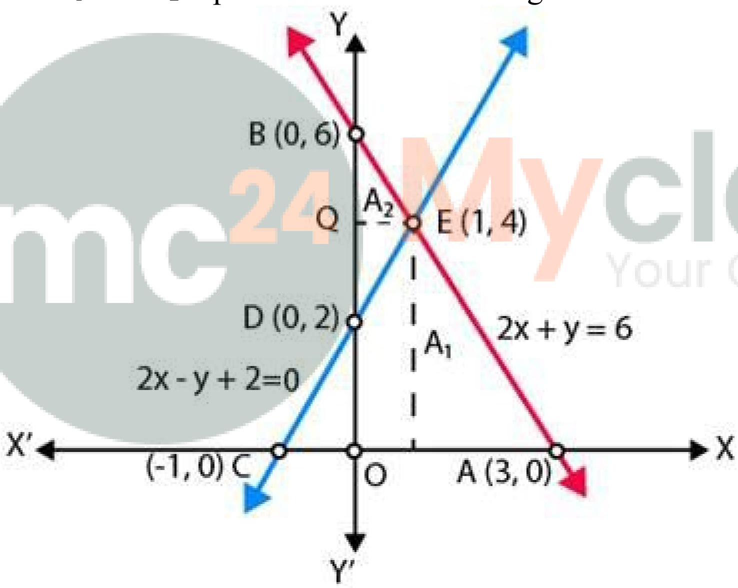
Table for equation $2x + y - 6 = 0$, for $x = 0$, $y = 6$, for $y = 0$, $x = 3$.

x	0	3
y	6	0

Table for equation $2x - y + 2 = 0$, for $x = 0$, $y = 2$, for $y = 0, x = -1$

x	0	-1
y	2	0

Let A_1 and A_2 represent the areas of triangles ACE and BDE respectively.



Let, Area of triangle formed with x -axis = T_1

T_1 = Area of $\triangle ACE = \frac{1}{2} \times AC \times PE$

$T_1 = \frac{1}{2} \times 4 \times 4 = 8$

And Area of triangle formed with y - axis = T_2

T_1 = Area of $\triangle BDE = \frac{1}{2} \times BD \times QE$

$T_1 = \frac{1}{2} \times 4 \times 1 = 2$

$T_1 : T_2 = 8:2 = 4:1$

Hence, the pair of equations intersect graphically at point E(1,4)

i.e., $x = 1$ and $y = 4$.

1. Determine, graphically, the vertices of the triangle formed by the lines
$y = x$, $3y = x$, $x + y = 8$

Solution:

Given linear equations are

y = x …(i)

3y = x …(ii)

and x + y = 8 …(iii)

Table for line y = x,

x	0	1	2
y	0	1	2

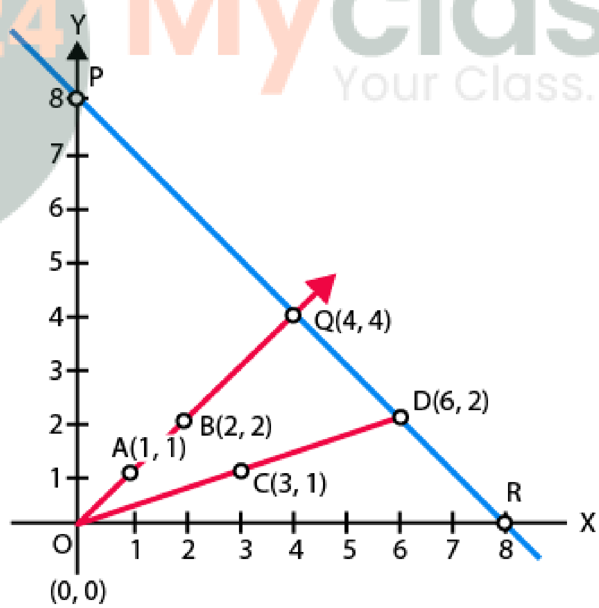
Table for line x = 3y,

x	0	3	6
y	0	1	2

Table for line x + y = 8

x	0	4	8
y	8	4	0

Plotting the points A (1, 1), B(2,2), C (3, 1), D (6, 2), we get the straight lines AB and CD. Similarly, plotting the point P (0, 8), Q(4, 4) and R(8, 0), we get the straight line PQR. AB and CD intersects the line PR on Q and D, respectively.



So, $\triangle OQD$ is formed by these lines. Hence, the vertices of the $\triangle OQD$ formed by the given lines are O(0, 0), Q(4, 4) and D(6,2).

2. Draw the graphs of the equations $x = 3$, $x = 5$ and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the x–axis.

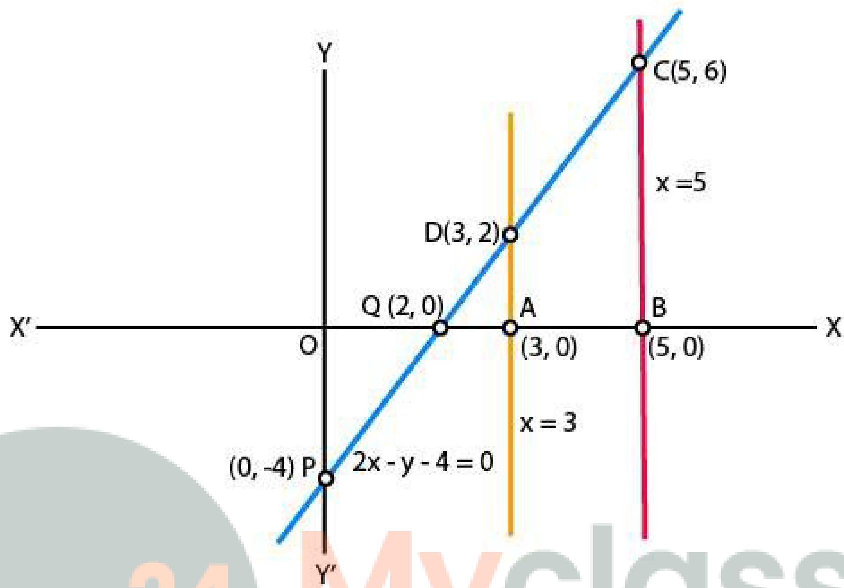
Solution:

Given equation of lines x = 3, x = 5 and 2x-y-4 = 0.

Table for line 2x - y - 4 = 0

x	0	2
y	-4	0

Plotting the graph, we get,



From the graph, we get,
AB = OB-OA = 5-3 = 2
AD = 2
BC = 6
Thus, quadrilateral ABCD is a trapezium, then,
Area of Quadrileral ABCD = ½ × (distance between parallel lines) = ½ × (AB) × (AD + BC)
= 8 sq units

3. The cost of 4 pens and 4 pencil boxes is Rs 100. Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.
Solution:

Let the cost of a pen and a pencil box be Rs x and Rs y respectively.
According to the question,
$4x + 4y = 100$
Or $x + y = 25$ …(i)
$3x = y + 15$
Or $3x - y = 15$ …(ii)
On adding Equation (i) and (ii), we get,
$4x = 40$
So, $x = 10$
Substituting x = 10, in Eq. (i) we get
$y = 25 - 10 = 15$

Hence, the cost of a pen = Rs. 10
The cost of a pencil box = Rs. 15

4. Determine, algebraically, the vertices of the triangle formed by the lines

$$3x - y = 3$$
$$2x - 3y = 2$$
$$x + 2y = 8$$

Solution:

3x - y = 2 …(i)
2x -3y = 2 …(ii)
x + 2y = 8 …(iii)
Let the equations of the line (i), (ii) and (iii) represent the side of a ΔABC.
On solving (i) and (ii),
We get,
[First, multiply Eq. (i) by 3 in Eq. (i) and then subtract]
(9x-3y)-(2x-3y) = 9-2
7x = 7
x = 1
Substituting x=1 in Eq. (i), we get
3×1-y = 3
y = 0
So, the coordinate of point B is (1, 0)
On solving lines (ii) and (iii),
We get,
[First, multiply Eq. (iii) by 2 and then subtract]
(2x + 4y)-(2x-3y) = 16-2
7y = 14
y = 2
Substituting y=2 in Eq. (iii), we get
x + 2 × 2 = 8
x + 4 = 8
x = 4
Hence, the coordinate of point C is (4, 2).
On solving lines (iii) and (i),
We get,
[First, multiply in Eq. (i) by 2 and then add]
(6x-2y) + (x + 2y) = 6 + 8
7x = 14
x = 2
Substituting x=2 in Eq. (i), we get
3 × 2 - y = 3
y = 3
So, the coordinate of point A is (2, 3).
Hence, the vertices of the ΔABC formed by the given lines are as follows,
A (2, 3), B (1, 0) and C (4, 2).

5. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw, and the remaining distance by bus.

Solution:

Let the speed of the rickshaw and the bus are x and y km/h, respectively.

Now, she has taken time to travel 2 km by rickshaw, $t_1 = (2/x)$ hr

Speed = distance/ time

she has taken time to travel remaining distance i.e., (14 - 2) = 12km

By bus $t_2 = (12/y)$ hr

By first condition,

$t_1 + t_2 = ½ = (2/x) + (12/y)$ … (i)

Now, she has taken time to travel 4 km by rickshaw, $t_3 = (4/x)$ hr

and she has taken time to travel remaining distance i.e., (14 - 4) = 10km, by bus = $t_4 = (10/y)$ hr

By second condition,

$t_3 + t_4 = ½ + 9/60 = ½ + 3/20$

$(4/x) + (10/y) = (13/20)$ …(ii)

Let $(1/x) = u$ and $(1/y) = v$

Then Equations. (i) and (ii) becomes

$2u + 12v = ½$ …(iii)

$4u + 10v = 13/20$…(iv)

[First, multiply Eq. (iii) by 2 and then subtract]

$(4u + 24v) – (4u + 10v) = 1–13/20$

$14v = 7/20$

$v = 1/40$

Substituting the value of v in Eq. (iii),

$2u + 12(1/40) = ½$

$2u = 2/10$

$u = 1/10$

$x = 1/u = 10$km/hr

$y = 1/v = 40$km/hr

Hence, the speed of rickshaw = 10 km/h

And the speed of bus = 40 km/h.