EXERCISE 2.4

If the polynomials $az^3 + 4z^2 + 3z – 4$ and $z^3 – 4z + a$ leave the same remainder when divided by $z – 3$, find the value of a.

Solution:

Zero of the polynomial,
$g_1(z) = 0$
$z-3 = 0$
$z = 3$
Therefore, zero of $g(z) = – 2a$
Let $p(z) = az^3+4z^2+3z-4$
So, substituting the value of $z = 3$ in $p(z)$, we get,
$p(3) = a(3)^3+4(3)^2+3(3)-4$
$\Rightarrow p(3) = 27a+36+9-4$
$\Rightarrow p(3) = 27a+41$
Let $h(z) = z^3-4z+a$
So, substituting the value of $z = 3$ in $h(z)$, we get,
$h(3) = (3)^3-4(3)+a$
$\Rightarrow h(3) = 27-12+a$
$\Rightarrow h(3) = 15+a$
According to the question,
We know that,
The two polynomials, p(z) and h(z), leaves same remainder when divided by z-3
So, $h(3)=p(3)$
$\Rightarrow 15+a = 27a+41$
$\Rightarrow 15-41 = 27a - a$
$\Rightarrow -26 = 26a$
$\Rightarrow a = -1$

1. The polynomial $p(x) = x^4 – 2x^3 + 3x^2 – ax + 3a – 7$ when divided by $x + 1$ leaves the remainder 19. Find the values of a. Also find the remainder when $p(x)$ is divided by $x + 2$.

Solution:

$p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7.$
Divisor = $x + 1$
$x + 1 = 0$
$x = -1$
So, substituting the value of $x = – 1$ in $p(x)$, we get,
$p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7.$
$19 = 1 + 2 + 3 + a + 3a – 7$
$19 = 6 - 7 + 4a$
$4a - 1 = 19$
$4a = 20$
$a = 5$
Since, $a = 5$.
We get the polynomial,

$p(x) = x^4 - 2x^3 + 3x^2 - (5)x + 3(5) – 7$
$p(x) = x^4 - 2x^3 + 3x^2 - 5x + 15 – 7$
$p(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$
As per the question,
When the polynomial obtained is divided by (x + 2),
We get,
$x + 2 = 0$
$x = - 2$
So, substituting the value of $x = – 2$ in p(x), we get,
$p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 5(-2) + 8$
$\Rightarrow p(-2) = 16 + 16 + 12 + 10 + 8$
$\Rightarrow p(-2) = 62$
Therefore, the remainder = 62.

2. If both $x – 2$ and $x – ½$ are factors of $px^2 + 5x + r$, show that $p = r$.
Solution:

Given, $f(x) = px^2+5x+r$ and factors are x-2, x - ½
$g_1(x) = 0$,
$x – 2 = 0$
$x = 2$
Substituting x = 2 in place of equation, we get
$f(x) = px^2+5x+r$
$f(2) = p(2)^2+5(2)+r=0$
$= 4p + 10 + r = 0$ … eq.(i)
$x – ½ = 0$
$x = ½$
Substituting x = ½ in place of equation, we get,
$f(x) = px^2+5x+r$
$f(½) = p(½)^2 + 5(½) + r = 0$
$= p/4 + 5/2 + r = 0$
$= p + 10 + 4r = 0$ … eq(ii)
On solving eq(i) and eq(ii),
We get,
$4p + r = – 10$ and $p + 4r = – 10$
Since the RHS of both the equations are same,
We get,
$4p + r = p + 4r$
$3p=3r$
$p = r$.
Hence Proved.

3. Without actual division, prove that $2x^4 – 5x^3 + 2x^2 – x + 2$ is divisible by $x^2 – 3x + 2$.
[Hint: Factorise $x^2 – 3x + 2$]
Solution:

x^2-3x+2
$x^2-2x-1x+2$

x(x-2)-1(x-2)
(x-2)(x-1)
Therefore,(x-2)(x-1)are the factors.

Considering (x-2),
x-2=0
x=2
Then, p(x) becomes,
p(x)=2
$p(x)=2x^4-5x^3+2x^2-x+2$
$p(2)=2(2)^4-5(2)^3+2(2)^2-2+2$
=32-40+8
= -40+40=0
Therefore, (x-2) is a factor.

Considering (x-1),
x-1=0
x=1
Then, p(x) becomes,
p(x)=1
$p(x)=2x^4-5x^3+2x^2-x+2$
$p(1)=2(1)^4-5(1)^3+2(1)^2-1+2$
=2-5+2-1+2
=6-6
=0
Therefore, (x-1) is a factor.