NCERT Solutions for Class-XI Physics

Chapter-8
NCERT Physics Class 11

1. You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?
If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (You can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why?

1. (a) No (b) Yes
Gravitational influence of matter on nearby objects cannot be screened by any means. This is because gravitational force unlike electrical forces is independent of the nature of the material medium. Also, it is independent of the status of other objects.
If the size of the space station is large enough, then the astronaut will detect the change in Earth's gravity (g).
Tidal effect depends inversely upon the cube of the distance while, gravitational force depends inversely on the square of the distance. Since the distance between the Moon and the Earth is smaller than the distance between the Sun and the Earth, the tidal effect of the Moon's pull is greater than the tidal effect of the Sun's pull.

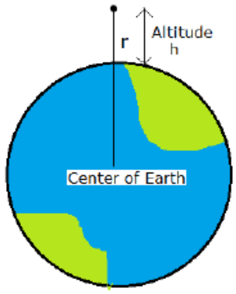
2. Choose the correct alternative:
Acceleration due to gravity increases/decreases with increasing altitude.
Acceleration due to gravity increases/decreases with increasing depth. (assume the earth to be a sphere of uniform density).
Acceleration due to gravity is independent of mass of the earth/mass of the body.
The formula $-G\ Mm\ (1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg\ (r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre

2. A.
Acceleration due to gravity decreases with increasing altitude, as it varies inversely to the square of distance from centre of earth and is given by relation

$$g = \frac{GM_e}{r^2}$$

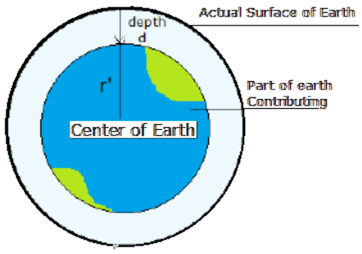
Where, g is the acceleration due to gravity
G is universal gravitational Constant
M_e is mass of Earth
r is distance of the point from center of earth (point must be on or above surface of earth not inside)

As can be seen in the figure



so as the distance from center of earth r or Altitude increases , the acceleration due to gravity decreases

B. Acceleration due to gravity decreases with increasing depth, as though distance of center of earth from the point is decreasing but mass of earth is also decreasing as less section of earth's mass will contribute to Gravity as can be seen in the figure



If at a depth d inside surface of earth, the acceleration due to gravity is given as g' = g(1–d/R) where, g' is acceleration due to gravity at a depth d inside surface of earth, R is the Radius of earth and g is acceleration due to gravity on surface of earth so as we can see as the depth inside surface d increases, value of d/R increases and (1-d/R) decreases and becomes less than 1, and hence we get g' < g inside surface of earth

C. Acceleration due to gravity is independent of mass of body as it is given by the relation $g = \frac{GM_e}{R^2}$ Where, g is the acceleration due to gravity

G is universal gravitational Constant

M_e is mass of Earth

R is the radius of earth as we can see it does not include any term of mass of body, so acceleration of gravity has same value for all bodies and is independent of mass of other body

D. The formula $-G\,Mm(1/r_2 - 1/r_1)$ is more accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth, as acceleration due to gravity varies with distance from centre from earth Potential energy assuming acceleration due to gravity to be constant is given as V = mgr Where V is the gravitational Potential Energy of body of mass m at a distance r from centre of earth, g is acceleration due to gravity So at distance r_1 from centre of earth gravitational potential energy will be $V_1 = mgr_1$ at distance r_2 from centre of earth gravitational potential energy will be $V_2 = mgr_2$ So difference in potential energy is $V_2 - V_1 = mgr_2 - mgr_1 = mg(r_2 - r_1)$ But as we know

Acceleration due to gravity decreases with increasing altitude, as it varies inversely to the square of distance from centre of earth and is given by relation $g = \frac{GM_e}{r^2}$

Where, g is the acceleration due to gravity

G is universal gravitational Constant

M_e is mass of Earth r is distance of the point from center of earth so g has different value at both the points i.e. the difference in potential energy is not accurate

The accurate relation for Gravitational Potential energy of a body of mass m at any point above surface of earth is given by relation

$V = -GMm/r$

Where V is the gravitational Potential Energy of body of mass m at a distance r from centre of earth and M is the mass of earth, So at distance r_1 from centre of earth gravitational potential energy will be

$V_1 = -GMm/r_1$ at distance r_2 from centre of earth gravitational potential energy will be

$V_2 = -GMm/r_2$

So difference in potential energy is

$V_2 - V_1 = -GMm/r_2 - (-GMm/r_1)$

$= -G\,Mm(1/r_2 - 1/r_1)$

This is more accurate formula for change in gravitational potential energy

3. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

3. Time taken by the Earth to complete one revolution around the Sun,

$T_e = 1$ year

Orbital radius of the Earth in its orbit, $R_e = 1$ AU

Time taken by the planet to complete one revolution around the Sun, $T_P = \frac{1}{2}T_e = -\frac{1}{2}$ year Orbital radius of the planet = R_p

From Kepler's third law of planetary motion, we can write:

$$\left(\frac{R_P}{R_e}\right)^3 = \left(\frac{T_P}{T_e}\right)^2$$

$$\frac{R_P}{R_e} = \left(\frac{T_P}{T_e}\right)^{\frac{2}{3}}$$

$$= \left(\frac{\frac{1}{2}}{1}\right)^{\frac{2}{3}} = (0.5)^{\frac{2}{3}} = 0.63$$

Hence, the orbital radius of the planet will be 0.63 times smaller than that of the Earth.

4. Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

4. If a body orbits around other heavier body due to gravitational force of attraction then we have a relation between mass of heavier body, time period of revolution and, radius of the orbit as $M = \frac{4\pi^2 R^3}{GT^2}$

Where M is the mass of heavier body, R is the radius of orbit, G is universal gravitational Constant and T is time period of revolution

Here we will consider two cases, motion of Satellite Io around Jupiter and motion of earth around sun

So for motion of Io around Jupiter we have

$$M_J = \frac{4\pi^2 R_{Io}^3}{GT_{Io}^2}$$

Where, M_j is the Mass of Jupiter

G is Universal gravitational constant

R_{Io} is radius of Io's orbit, we are given

$R_{Io} = 4.22 \times 10^8$ m

T_{Io} is the time period of revolution of Io around Jupiter

$T_{Io} = 1.769$ days

So for motion of Earth around Sun we have

$$M_s = \frac{4\pi^2 R_e^3}{GT_e^2}$$

Where, M_s is the Mass of sun

G is Universal gravitational constant

R_e is radius of Earth's orbit

R_e = **1 AU = 1.496 × 10^{11} m**

T_e is the time period of revolution of Earth around sun

$T_e = 365.25$ days

Diving equation of mass of sun with equation of mass of Jupiter to compare

$$\frac{M_s}{M_j} = \frac{4\pi^2 R_e^3}{GT_e^2} \times \frac{GT_{Io}^2}{4\pi^2 R_{Io}^3} = \left(\frac{R_e}{R_{Io}}\right)^3 \left(\frac{T_{Io}}{T_e}\right)^2$$

So putting values of R_e , R_{Io}, T_{Io}, T_e in above equation we get

$$\frac{M_s}{M_j} = \left(\frac{1.496 \times 10^{11} m}{4.22 \times 10^8 m}\right)^3 \left(\frac{1.769}{365.25}\right)^2$$

NOTE: Time periods are not converted to SI units second, but are in years , but would not make any difference to result because we are taking ration and units and converting factors are going to be cancelled out ultimately

Solving above equation we get

$$\frac{M_s}{M_j} \approx 1000$$

Or we can say

$M_s \approx 1000 M_j$

i.e. mass of Sun is nearly 1000 times mass of Jupiter

5. Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 105 ly.

5. There are huge Stars in galaxy which together constitute a very huge mass and centre point of all concentrated mass galactic centre, we assume Star revolve around galactic centre in the same way in which planets revolve around sun we can assume all starts in milky way to constitute one heavenly body assuming all the mass to be concentrated at a single point Galactic centre and other heavenly bodies like stars revolve around galactic centre so we can treat distance of star from milky way as the orbital radius of revolving star

We know If a body orbits around other heavier body due to gravitational force of attraction then we have a relation between mass of heavier body, time period of revolution and, radius of the orbit as

$$M = \frac{4\pi^2 R^3}{GT^2}$$

Where M is the mass of heavier body, R is the radius of orbit, G is universal gravitational Constant

$G = 6.67 \times 10^{-11}\ Nm^2Kg^{-2}$ and T is time period of revolution

So rearranging above equation we get time period of revolution as $T = \left(\frac{4\pi^2 R^3}{GM}\right)^{\frac{1}{2}}$

We are give total mass of Mass of Galaxy as

$M = 2.5 \times 10^{11}$ solar mass

Now 1 solar mass is mass of sun which is

$M_s = 2 \times 10^{30}$ Kg

So total mass of the galaxy is

$M = 2.5 \times 10^{11} \times M_s$

$= 2.5 \times 10^{11} \times 2 \times 10^{30}$ Kg

$= 5 \times 10^{41}$ Kg we are given distance of star from galactic center which is Radius of Milky way or orbital radius for star that is $R = 50000$ ly $= 5 \times 10^4$ ly

We know that one light year is distance travelled by light in one year And distance is given as

$S = V \times t$

Where S is the distance covered in time t moving at a speed V Speed of light is

$V = 3 \times 10^8$ m/s

Converting time of 1 year into seconds

1 year = 365.25 days

1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

So we get time of 1 year as

$t = 365.25 \times 24 \times 60 \times 60$ s

so we get distance of 1 light year as

$S = 3 \times 10^8$ m/s $\times 365.25 \times 24 \times 60 \times 60$ s

i.e 1 light year $= 9.46 \times 10^{15}$ m Therefore the radius of milky way is

$R = 5 \times 10^4 \times 9.46 \times 10^{15} = 4.73 \text{ x } 10^{20}$ m

So putting values of R, M , G in equation to find time period

$$T = \left(\frac{4\pi^2 R^3}{GM}\right)^{\frac{1}{2}}$$

$$T = \left(\frac{4 \times \pi^2 \times (4.73 \times 10^{20} \text{m})^3}{6.67 \times 10^{-11} \text{Nm}^2\text{Kg}^{-2} \times 5 \times 10^{41} \text{Kg}}\right)^{\frac{1}{2}}$$

$= 4.246 \times 10^{15}$ s

Converting into years

We already know

1 year = 365.25 × 24 × 60 × 60 s

So 1 s = 1/(365.25 × 24 × 60 × 60) Year

So we get time period in the year as

$$T = \frac{4.246 \times 10^{15} \text{s}}{365.25 \times 24 \times 60 \times 60} \text{year} = 1.34 \times 10^8 \text{Years}$$

so star will take 1.34×10^8 Years to complete one revolution

6. Choose the correct alternative:

A. If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

B. The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

6. Kinetic energy

Less

Total mechanical energy of a satellite is the sum of its kinetic energy (always positive) and potential energy (may be negative). At infinity, the gravitational potential energy of the satellite is zero. As the Earth-satellite system is a bound system, the total energy of the satellite is negative.

Thus, the total energy of an orbiting satellite at infinity is equal to the negative of its kinetic energy.

An orbiting satellite acquires a certain amount of energy that enables it to revolve around the Earth. This energy is provided by its orbit. It requires relatively lesser energy to move out of the influence of the Earth's gravitational field than a stationary object on the Earth's surface that initially contains no energy.

7. Does the escape speed of a body from the earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched?

7. Escape speed is the minimum speed a body must acquire in order to move out of earth's gravitational field or escape from earth's influence. This happens when total energy body becomes zero or positive and is in the unbound state.

(a) No, Escape speed does not depend upon the mass of the body as escape speed is given by the relation

$$V_S = \sqrt{\frac{2GM_e}{R}}$$

Here, V_S is escape velocity of a body at a distance of R from the centre of earth G is universal Gravitational Constant, M_e is the mass of earth

So as we can see the relation of escape velocity is independent of the mass of body itself.

(b) No, Escape speed does not depend upon location from where it is projected assuming the condition that earth is a perfect sphere and radius of earth's surface is almost same everywhere, because then according to relation of escape velocity, the distance from centre of earth R will be a constant value irrespective of latitude and longitude hence the escape speed is invariant of location.

(c) No, Escape speed does not depend upon the direction of projection as we take any direction value of gravitational Constant G, the mass of earth M_e and Radius at the surface of earth R will be constant so escape speed will be constant. Though if we take earth's spin into account as well then the value will depend on direction though we are neglecting it here.

(d) Yes, Escape speed depends on the height of the location from where the body is launched as we can see in the relation escape velocity varies inversely to the square root of the distance from the centre of the earth

$$V_s \propto \frac{1}{\sqrt{R}}$$

Where V_s is escaped speed and R is the distance from the centre of the earth

Now as the height of body from the surface of the earth increases its distance from the centre of earth increases and Escape velocity decreases.

8. A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a)linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

8. No

No

Yes

No

No

Yes

Angular momentum and total energy at all points of the orbit of a comet moving in a highly elliptical orbit around the Sun are constant. Its linear speed, angular speed, kinetic, and potential energy varies from point to point in the orbit.

9. Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem?

9. (b) swollen face, (c) headache, (d) orientational problem are likely to afflict an astronaut in space

Swollen feet is a situation when due to excess body mass, resulting in excessive body weight applies force on feet which results in causing fluid to build up in feet, legs and ankles, but in space this problem will not afflict an astronaut in space because, there is no gravity so there will be no force on feet due to body weight as body weight will be zero though mass will be same we know weight is given as $W = mg$

Where W is the weight of a body of mass m and g is acceleration due to gravity

In space there is no gravity so

$g = 0$

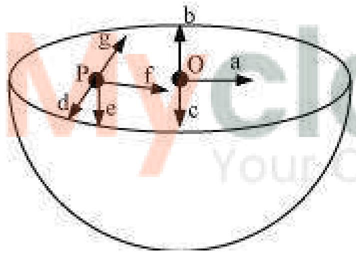
i.e. body weight is zero so there is no effect of swollen feet.

as there is no air in space and hence no air pressure, blood circulation with an increase in body and face as there is no external pressure to counter it, so it may cause the face to swell and also lead to a headache as blood circulation in brain too increases

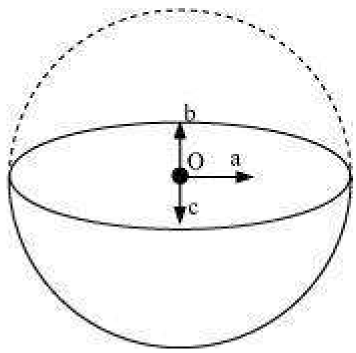
Orientation is a function of mind involving awareness place time person, space also has a definite orientation (definite position of one point with respect to another) so if a person has orientation problem i.e. brain is not working properly regarding positions and direction this will definitely afflict the person in space

10. Choose the correct answer from among the given ones:

The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 8.12) (i) a, (ii) b, (iii) c, (iv) O.



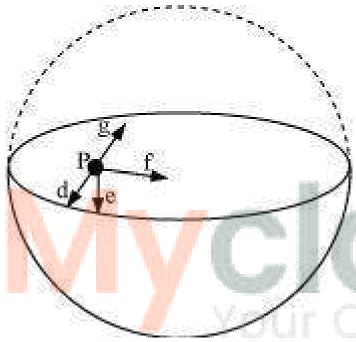
10. Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient $\left(\frac{dv}{dr}\right)$ is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric. If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle located at centre O will be in the downward direction.



Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at centre O of the given hemispherical shell has the direction as indicated by arrow c.

11. Choose the correct answer from among the given ones: For the problem 8.10, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

11. (ii) Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential $\left(\frac{dv}{dr}\right)$ gradient is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric. If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle at an arbitrary point P will be in the downward direction.

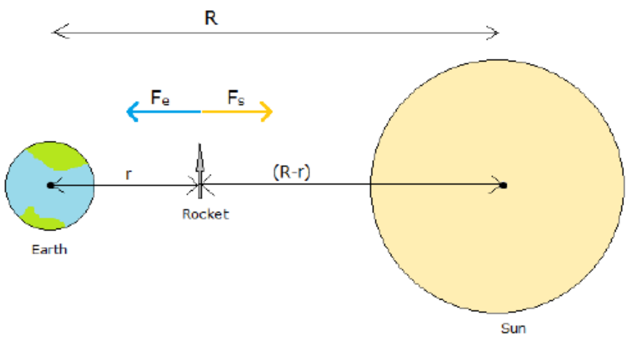


Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at an arbitrary point P of the hemispherical shell has the direction as indicated by arrow e.

12. A rocket is fired from the earth towards the sun. At what distance from the earth's center is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

12. The net gravitational force on the rocket will be zero when attractive gravitational force on Rocket due to sun is equal in magnitude to attractive gravitational force by Earth, so it is between earth and sun, suppose distance between earth and sun or the orbital radius is R, and rocket is at a distance r from earth so it is at (R-r) distance from sun and attractive pull balances attractive pull of earth

The situation has been depicted in the figure



The gravitational force on Rocket due to sun F_s is equal in magnitude to attractive gravitational force by Earth F_e and opposite in direction as can be seen in the figure

we know gravitational force on a body is given as

$$F = \frac{Gm_1m_2}{r^2}$$

Where F is the gravitational force

G is universal gravitational Constant

m_1 is mass of the first body

m_2 is the mass of the second body

and r is the distance between the two bodies

Now gravitation force on rocket due to earth will be

$$F_e = \frac{GM_eM_r}{r^2}$$

Where M_e and M_r are masses of earth and rocket and r is a separation between them

Similarly, gravitation force on rocket due to Sun will be

$$F_s = \frac{GM_sM_r}{(R-r)^2}$$

Where M_s and M_r are masses of Sun and rocket and separation between Sun and rocket is (R – r)

Since both forces should be equal in magnitude, equating both

$F_e = F_s$

$$\text{i.e.} \frac{GM_eM_r}{r^2} = \frac{GM_sM_r}{(R-r)^2}$$

solving and canceling terms we get

$$\frac{(R-r)^2}{r^2} = \frac{M_s}{M_e}$$

Or we can say

$$\frac{R-r}{r} = \left(\frac{M_s}{M_e}\right)^{\frac{1}{2}}$$

We are given

Mass of the sun, $M_s = 2\times10^{30}$ kg

Mass of the earth, $M_e = 6\times10^{24}$ kg

The distance between Earth and Sun or orbital Radius

$R = 1.5 \times 10^{11}$ m

So putting these values to find the distance between earth and rocket r

$$\frac{1.5\times10^{11}\,\text{m}-\text{r}}{\text{r}}=\left(\frac{2\times10^{30}\,\text{Kg}}{6\times10^{24}\,\text{Kg}}\right)^{\frac{1}{2}}$$

$577.3r = 1.5 \times 10^{11}$ m – r

$578.3r = 1.5 \times 10^{11}$ m

$r = (1.5 \times 10^{11}$ m)/ $578.3 = 2.59 \times 10^3$ m

so the rocket is at a distance of 2.59×10^3 m from earth's centre

13. How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

13. Orbital radius of the Earth around the Sun, r = 1.5 × 1011 m

Time taken by the Earth to complete one revolution around the Sun,

T = 1 year = 365.25 days

= 365.25 × 24 × 60 × 60 s

Universal gravitational constant, $G = 6.67 \times 10^{-11}$ $Nm^2\ kg^{-2}$

Thus, mass of the Sun can be calculated using the relation,

$$M=\frac{4\pi^2r^3}{GT^2}$$

$$=\frac{4\times(3.15)^2\times(1.5\times10^{11})^3}{6.67\times10^{-11}\times(365.25\times24\times60\times60)^2}$$

$$=\frac{133.24\times10}{6.64\times10^4}=2.0\times10^{30}\,\text{kg}$$

Hence, the mass of the Sun is 2×10^{30} kg.

14. A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50 × 108 km away from the sun?

14. Earth and Saturn both revolve in definite orbits around sun due to gravitational force of attraction due to sun now when a planet revolves in orbit around sun due to attractive gravitational force between planet and sun, from Kepler's third law of planetary motion we know that square of time period one complete rotation of planet T around sun is proportional to cube of mean distance (average distance) between planet and sun R

$T^2 \propto R^3$

Le the Time taken by the earth for one complete revolution be T_e

T_e = 1 Year

Let mean distance of the earth from the sun or orbital radius be R_e

Time is taken by the Saturn for one complete revolution or its time period of revolution around the sun be T_s

We are given that a Saturn's year is 29.5 times the earth year i.e. time period of Saturn is 29.5 times that of earth

i.e. $T_s = 29.5\ T_e$

Let, the orbital radius of this Saturn be R_s

Now, according to the Kepler's third law of planetary motion, we have

$T_e^2 \propto R_e^3$

And

$T_S^2 \propto R_s^3$

sing both equations we get the relation

$\frac{T_e^2}{T_p^2} = \frac{R_e^3}{R_s^3}$;

$\left(\frac{R_s}{R_e}\right)^3 = \left(\frac{T_s}{T_e}\right)^2$

Or we can say $\left(\frac{R_s}{R_e}\right)^3 = \left(\frac{T_s}{T_e}\right)^2$

Simplifying we get the relation for the orbital radius of Saturn as

$R_s = R_e\left(\frac{T_p}{T_e}\right)^{\frac{2}{3}}$

now we have

$T_s = 29.5\ T_e$

i.e. $T_s/T_e = 29.5$

i.e. radius of planet is

$R_p = R_e(29.5)^{\frac{2}{3}} = 9.54R_e$

Putting the value of the orbital radius of the earth

$R_e = 1.50 \times 10^8$ km

we get the distance of Saturn from the sun as

$R_p = 9.54 \times 1.50 \times 10^8$ km $= 1.43 \times 10^9$ Km $= 1.43 \times 10^{12}$ m

So orbital radius of Saturn is 1.43×10^9 Km or we can say it is at a distance of 1.43×10^{12} m from sun

15. A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

15. Weight of the body, W = 63 N

Acceleration due to gravity at height h from the Earth's surface is given by the relation:

$g' = \frac{g}{\left(\frac{1+h}{R_e}\right)^2}$

When

g = Acceleration due to gravity on the Earth's surface

Re = Radius of the Earth

For h = $\frac{R_e}{2}$

$g' = \frac{g}{\left(1+\frac{R_e}{2\times R_e}\right)^2} = \frac{g}{\left(1+\frac{1}{2}\right)^2} = \frac{4}{9}g$

Weight of a body of mass m at height h is given as:

W' = mg'

$$= m \times \frac{4}{9} g = \frac{4}{9} \times mg$$

$$= \frac{4}{9} W$$

$$= \frac{4}{9} \times 63 = 28N$$

16. Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?

16. The weight of a body is the force acting one it due to earth's gravity and is the product of mass and acceleration due to gravity is given as

$W = mg$

Where W is the weight of a body of mass m on the surface of the earth, g is acceleration due to gravity on the surface of the earth

Now as the body goes inside surface of earth towards the centre of earth acceleration due to gravity decreases, so the weight of the body also decreases i.e.

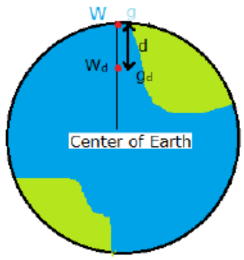
$W_d = mg_d$

Where W_d is the weight of a body of mass m kept at a depth d inside the surface of the earth, Let g_d be the acceleration due to gravity at a at a depth d inside the surface of the earth. Now acceleration due to gravity at a depth d inside the surface of the earth is given by the relation

$$g_d = g\left(1 - \frac{d}{R}\right)$$

Where g_d is the acceleration due to gravity at a depth d inside the surface of the earth, g is acceleration due to gravity on earth's surface and R is Radius of the Earth

The situation has been shown in figure



Now Let g_d be the acceleration due to gravity half way down to the centre of the earth and g be the acceleration due to gravity on the surface of the earth.

If R is the radius of the Earth and mass in half way down the earth then we have the depth of body from the surface of earth

$d = R/2$

Then using

$$g_d = g\left(1 - \frac{d}{R}\right)$$

We get

$$g_d = g\left(1 - \frac{\left(\frac{R}{2}\right)}{R}\right) = g\left(1 - \frac{R}{2R}\right)$$

g_d = g/2

i.e. acceleration due to gravity half way down the surface of the earth is half of that on the surface of the earth

so the weight of the body at this depth will be

$W_d = mg_d$

$W_d = m(g/2) = ½ mg$

i.e. $W_d = ½ W$

now we are given the weight of the body on the surface of the earth as

W = 250 N

i.e. we have

$W_d = 250/2$ N

$W_d = 125$ N

So, the weight of the body at a depth equal to half the radius of the earth is 125 N.

17. A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0 × 10^{24} kg; mean radius of the earth = 6.4 × 10^6 m; G = 6.67 × 10^{-11} N m2 kg^{-2}.

17. 8 × 10^6 m from the centre of the Earth

Velocity of the rocket, v = 5 km/s = 5 × 10^3 m/s

Mass of the Earth, $M_e = 6.0 \times 10^{24}$ kg

Radius of the Earth, $R_e = 6.4 \times 10^6$ m

Height reached by rocket mass, m = h

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e}\right)$$

At highest point h,

v = 0

And, Potential energy = $\frac{GM_e m}{R_e + h}$

Total energy of the rocket $= 0 + \left(-\frac{GM_e m}{R_e + h}\right) = \frac{GM_e m}{R_e + h}$

From the law of conservation of energy, we have

Total energy of the rocket at the Earth's surface = Total energy h

$$\frac{1}{2}mv^2 + \left(-\frac{\Gamma M_e m}{R_e}\right) = -\frac{GM_e m}{R_e + h}$$

$$\frac{1}{2}v^2 = GM_e\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right)$$

$$= GM_3\left(\frac{R_e + h - R_e}{R_e(R_e + h)}\right)$$

$$\frac{1}{2}v^2 = \frac{GM_eh}{R_e(R_e + h)} \times \frac{R_e}{R_e}$$

$$\frac{1}{2} \times v^2 = \frac{gR_eh}{R_e + h}$$

Where $g = \frac{GM}{R_e^2} = 9.8$ m/s^2 (Acceleration due to gravity on the Earth's surface)

$$\therefore v^2(R_e + h) = 2gR_eh$$

$$v^2R_e = h(2gR_e - v^2)$$

$$h = \frac{R_ev^2}{2gR_e - v^2}$$

$$= \frac{6.4 \times 10^6 \times (5 \times 10^3)^2}{2 \times 9.8 \times 6.4 \times 10^6 - (5 \times 10^3)^2}$$

$$h = \frac{6.4 \times 25 \times 10^{12}}{100.44 \times 10^6} = 1.6 \times 10^6 \text{m}$$

Height achieved by the rocket with respect to the centre of the Earth

$= R_e + h$

$= 6.4 \times 10^6 + 1.6 \times 10^6$

$= 8.0 \times 10^6$m

18. The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

18. Escape speed is that after attaining which a body moves just out of the earth's Gravitational influence

As body will be out of earth's influence when its total energy is Zero or positive and we know the Total energy of a body is the sum of kinetic energy and potential energy

T = K + U

Where T is the total energy, U is potential energy and K is kinetic energy kinetic energy of a body which is always positive and depends upon the speed of the body, potential energy is negative and decreases as we move away from orbiting the planet and at an infinite distance from the planet

Kinetic Energy is given as

$$K = \frac{1}{2}mv^2$$

Where K is the kinetic energy of a body of mass m, moving with velocity v

The potential energy of a body above the surface of the earth is given as

U = –GMm/R

Where U is the gravitational Potential Energy of body of mass m at a distance R from the centre of the earth and M is the mass of earth G is universal gravitational Constant

If Body have to move out of Earth's influence its total energy should be positive i.e.

$T \geq 0$

$$\text{Or} \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) \geq 0$$

Let m be the mass of the projectile and V_e be the escape speed from surface of earth. So kinetic energy of a body at escape speed K_e is

$K_e = \frac{1}{2} m V_e^2$

when distance of body from centre of earth will be equal to radius of earth, Potential energy of particle at surface of earth is

$U = -GMm/r$

i.e. for a body to just escape total energy should be zero i.e.

$$\frac{1}{2}mV_e^2 + \left(-\frac{GMm}{r}\right) = 0$$

i.e. $K_e = \frac{1}{2} m V_e^2 = -(-GMm/r)$

or $U = -K_e$

so potential energy of a body at surface of earth is equal to negative of kinetic energy at escape velocity

We are given initial speed V_i is three times escape speed i.e.

$V_i = 3Ve$

Let m be the mass of the projectile, then its initial kinetic energy will be

$K_i = \frac{1}{2} mV_i^2 = \frac{1}{2} m(3V_e)^2$

$=9(\frac{1}{2} m V_e^2) = 9 K_e$

So initial kinetic energy of projectile is 9 times kinetic energy at escape speed

Now initial potential energy of projectile when it is at surface of earth will be

$U_i = -GMm/R$

We know it will be equal to negative of the kinetic energy of the same particle at escape velocity

$U_i = -K_e$

So total initial energy of projectile will be

$T_i = K_i + U_i$

i.e. $T_i = 9 K_e + (-K_e) = 8 K_e$

so total initial energy of projectile will be 8 times its kinetic energy at escape velocity

now finally when projectile will be at an infinite distance, its potential energy will be zero

$U_f = 0$

And let us assume particle of mass m has gained a velocity V_f, so the final kinetic energy of projectile will be

$K_f = \frac{1}{2} mV_f^2$

So total final energy will be

$T_f = K_f + U_f$

i.e. $T_f = \frac{1}{2} mV_f^2$

using the law of conservation of energy we know total initial energy must be equal to total final energy so we have

$T_i = T_f$

Or we can say

$8(K_e) = ½\, mV_f^2$

$8 \times (½\, mV_e^2) = ½\, mV_f^2$

On solving we get the relation between final speed V_f and Escape speed velocity V_i as

$V_f^2 = 8V_e^2$

Or $V_f = \sqrt{8}V_e$

Now we know escape velocity is

$V_e = 11.2$ km/s $= 11.2 \times 10^3$ m/s

the final velocity of projectile V_f far away from earth will be

$V_f = \sqrt{8} \times 11.2 \times 10^3 \text{ms}^{-1}$

i.e. $V_f = 31.68 \text{ X } 10^3$ m/s = 31.68 km/s

so final velocity of Projectile at distance far away from earth is 31.68 km/s

19. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^6 m; G = 6.67×10^{-11} N m2 kg^{-2}.

19. Mass of the Earth. M = $6.0 \times 10 \times 10^{24}$ kg

Mass of the satellite, m = 200 kg

Radius of the Earth, $R_e = 6.4 \times 10^6$ m

Universal gravitational constant, G = 6.67×10^{-11} Nm^2 kg^{-2}

Height of the satellite, h = 400 km = 4×10^5 m = 0.4×10^6 m

Total energy of the satellite at height h $= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e + h}\right)$

Orbital velocity of the satellite, v = $\sqrt{\frac{GM_e}{R_e + h}}$

Total energy of height, h $= \frac{1}{2}m\left(\frac{GM_e}{R_e + h}\right) - \frac{GM_e m}{R_e + h} = -\frac{1}{2}\left(\frac{GM_e m}{R_e + h}\right)$

The negative sign indicates that the satellite is bound to the Earth. This is called bound energy of the satellite.

Energy required to send the satellite out of its orbit = – (Bound energy)

$$= \frac{1}{2}\frac{GM_e m}{(R_e + h)}$$

$$= \frac{1}{2} \times \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{(6.4 \times 10^6 + 0.4 \times 10^6)}$$

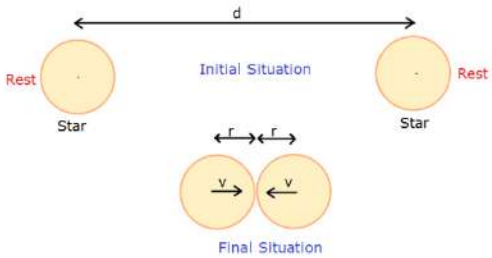
$$= \frac{1}{2} \times \frac{6.67 \times 6 \times 2 \times 10}{6.8 \times 10^6} = 5.9 \times 10^9 \text{ J}$$

20. Two stars each of one solar mass (= 2×10^{30} kg) are approaching each other for a head on collision. When they are a distance 109 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 104 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

20. Now let us assume both stars to be initially at rest and come towards each other due to the gravitational force of attraction and as they come closer their speed keeps on increasing, suppose they are initially at a distance of d

$d = 10^9$ km $= 10^{12}$ m

For head-on collision, the least distance between the stars will be equal to twice of their radius. The Initial situation and when they are just going to collide has been displayed in the figure below



For head-on collision, the least distance between the stars will be equal to twice of their radius, we are given the radius of each star as $r = 10^4$ km $= 10^7$ m so least distance or final between the stars will be

$d' = 2r = 2 \times 10^7$ m

we will use the law of conservation of energy

The total energy of a body is the sum of kinetic energy and potential energy

$T = K + U$

Where T is the total energy, U is potential energy and K is kinetic energy

We know Kinetic Energy is given as

$K = \frac{1}{2}mv^2$

Where K is the kinetic energy of a body of mass moving with speed v and potential energy is given as

$U = -Gm_1m_2/R$

Where U is the potential Energy of a body of mass m_1 at a point at a distance R from a center of mass of Body of mass m_2, G is universal Gravitational constant

Initially, Both the Stars are assumed to be at rest i.e their speed is

$v = 0$

so initial Kinetic energy of both the stars will be zero so the total kinetic energy of the system consisting two stars will also be zero

i.e. $K_i = 0$

Initially, potential energy of the system will have a maximum value when the separation between stars will be maximum

now masses of both the stars are equal i.e.

$m_1 = m_2 = m = 2\times10^{30}$ kg

value of the universal gravitational constant is

$G = 6.67 \times 10^{-11}$ Nm^2Kg^{-2}

And the initial separation between the stars is

$R = d = 10^{12}$ m

so the initial gravitational potential energy of the system U_i will be

$$U_i = -\frac{G\times m\times m}{d} = -\frac{6.67\times10^{-11}\,Nm^2Kg^{-2}\times\left(2\times10^{30}\,Kg\right)^2}{10^{12}\ m}$$

$= -26.68\times 10^{37}\,J$

So total initial energy of the system will be

$T_i = U_i + K_i$

i.e. $T_i = -26.68 \times 10^{37}$ J + 0 J

= -26.68×10^{37} J

Now finally let us assume both stars gained speed v moving towards each other and are about to collide, so the final kinetic energy of the system will be equal to the sum of kinetic energy of both the stars i.e.

$K_f = ½ mv^2 + ½ mv^2 = mv^2$

Where m is the mass of each star, we know

$m = 2\times10^{30}$ kg

so the final kinetic energy of the system will be

$K_f = 2\times10^{30}\times v^2$

Finally when stars will come close and separation between them will be minimum potential energy will also be at its minimum value

We know final separation between the stars is

$R = d' = 2 \times 10^7$ m

now masses of both the stars are equal i.e.

$m_1 = m_2 = m = 2\times10^{30}$ kg

value of the universal gravitational constant is

$G = 6.67 \times 10^{-11}\ Nm^2Kg^{-2}$

so Final gravitational potential energy of the system U_f will be

$$U_f = -\frac{G\times m\times m}{d'} = -\frac{6.67\times10^{-11}\,Nm^2Kg^{-2}\times\left(2\times10^{30}\,Kg\right)^2}{2\times10^{7}\ m}$$

$= -13.34 \times 10^{42}$ J

So total final energy of the system will be

$T_f = U_f + K_f$

i.e. $T_f = -13.34 \times 10^{42}\ J + 2\times10^{30}\times v^2$

NOTE : while calculating total kinetic energy of the system we added individual kinetic energies of both the stars but we did not do this for potential energy as potential energy of a body due to another body is the potential energy of the system and taking any body as reference(P.E. of star 1 w.r.t star 2 or P.E. of star 2 w.r.t. star 1) when we calculate potential energy value is same now as we can observe potential energy of the system has been lost and is converted to kinetic energy but according to the law of conservation of energy total energy will be same i.e.

$T_i = T_f$

So equating values

$-26.68 \times 10^{37}\ J = -13.34 \times 10^{42}\ J + 2\times10^{30}Kg \times v^2$

Solving further

$2 \times 10^{30}Kg \times v^2 = 13.34 \times 10^{42}\ J - 0.00026 \times 10^{42}\ J$

$v^2 = 6.66 \times 10^{12}\ m^2s^{-2}$

i.e $v = \sqrt{6.66 \times 10^{12} m^2 s^{-2}} = 2.58 \times 10^6 ms^{-1}$

so we get final speed of both the stars as 2.58×10^6 ms^{-1}

21. Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centers of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

21. Let us consider two spheres each of mass M and radius r are placed at a distance d apart from each other on a horizontal table,

Here a mass of both the spheres is

M = 100 Kg

Here Radius of both the spheres is

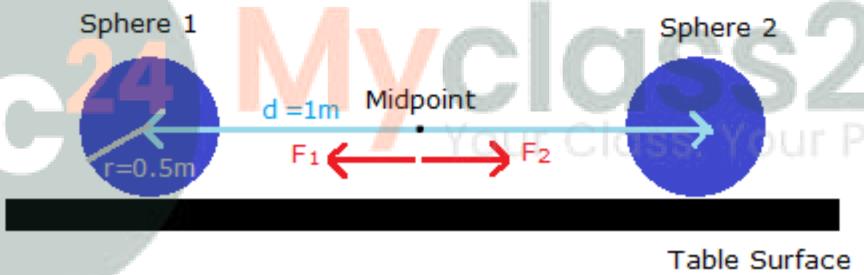
r = 0.10 m

The separation between both the spheres is

d = 1.0 m

suppose sphere 1 is on left side of table and Sphere 2 is on right side of table, suppose a body is placed at the midpoint of line joining two spheres, Since gravitational force is always attractive in nature so let Gravitational Force due to sphere 1 be F_1 and it will be in left direction, and force due to sphere 2 is F_2 and will act on right side

The situation has been shown in the figure



We know gravitational force on a body is given as

$$F = \frac{GMm}{R^2}$$

Where F is the gravitational force

G is universal gravitational Constant

$G = 6.67 \times 10^{-11}$ Nm^2Kg^{-2}

M is mass of the first body m is the mass of the second body and R is the distance between the two bodies now here if Body is kept at the midpoint of the line joining two spheres then the separation between a body at the midpoint and the two spheres will be same and equal to half the distance between two spheres suppose R_1 is a distance of the first sphere from midpoint and R_2 is a distance of the second sphere from midpoint then we have $R_1 = R_2 = R = d/2 = ½ = 0.5m$ now we get that magnitude of force on a body kept at midpoint by sphere 1 and sphere 2 is same as for both the forces F_1 and F_2 , mass of first body(Sphere) M, mass of the second body(body at midpoint) m, separation between both the bodies R is same and Universal gravitational constant is same , i.e. magnitude of forces $F_1 = F_2 = GMm/R^2$ Now magnitude of both the forces will be exactly same but directions will be opposite, and force is a vector quantity so forces will be added vectorially and we know Force of equal magnitude and opposite direction result in Zero as both cancel out each other, Magnitude of the net force is $F = F_1 - F_2 = 0$ N

So net force at the centre is 0 N Now Gravitational potential at a point is given by the relation

$V = -GM/R$

Where V is the potential of a point at a distance R from a Body of mass M, G is universal Gravitational constant

Potential is a scalar quantity so it does not have a direction and is added directly not net potential at the centre will be

$V = V_1 + V_2$

Where V_1 is the potential due to the first body and V_2 is the potential due to the second body

We know potential due to the first sphere will be given as

$V_1 = -GM_1/R_1$

Where M_1 is the Mass of the first sphere

$M_1 = M = 100$ Kg

R_1 is a distance of the first sphere from Midpoint

$R_1 = R = 0.5$ m

G is universal gravitational Constant

$G = 6.67 \times 10^{-11}\ Nm^2Kg^{-2}$

Now putting the values in above equation we get

$$V_1 = -\frac{6.67\times10^{-11}\,Nm^2Kg^{-2}\times100Kg}{0.5m} = -1.334\times10^{-8}\,Jkg^{-1}$$

similarly, potential due to the second sphere will be given as

$V_2 = -GM_2/R_2$

Where M_2 is the Mass of the Second sphere

$M_2 = M = 100$ Kg

R_2 is a distance of the Second sphere from Midpoint

$R_2 = R = 0.5$ m

G is universal gravitational Constant

$G = 6.67 \times 10^{-11}\ Nm^2Kg^{-2}$

Now putting value we get

$$V_2 = -\frac{6.67\times10^{-11}\,Nm^2Kg^{-2}\times100Kg}{0.5m} = -1.334\times10^{-8}\,Jkg^{-1}$$

So net potential at mid point is

$V = V_1 + V_2$

$= -1.334 \times 10^{-8}\ Jkg^{-1} + (-1.334 \times 10^{-8}\ Jkg^{-1})$

$= -2.668 \times 10^{-8}\ Jkg^{-1}$

If a body is placed at Midpoint it will be in equilibrium as net force acting on it will be Zero , but it be in unstable equilibrium as if it is displaced slightly to left or right it will be pulled towards the sphere which is at less distance from it and not come back to its mean position this will happen because gravitational force of attraction is inversely proportional to distance so as distance between the two bodies decrease force of attraction increase and vice versa, so on even slight displacement force on body due to one sphere will increase and decrease due the other sphere which is at a greater distance and it will be pulled towards body at lesser distance and never come back to original position such an equilibrium condition is called unstable equilibrium.