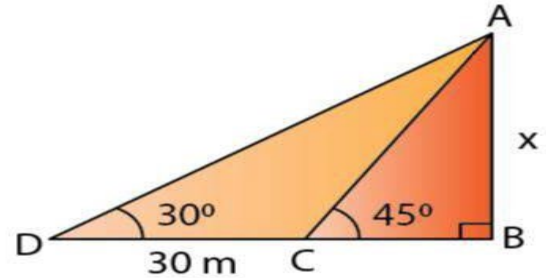
Exercise 22(B)

1. In the figure, given below, it is given that AB is perpendicular to BD and is of length X metres. DC = 30 m, ∠ADB = 30º and ∠ACB = 45º. Without using tables, find X.

Solution:

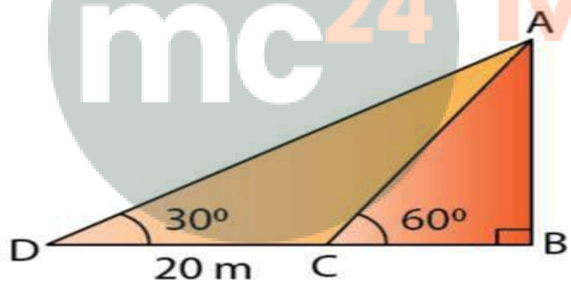


In ΔABC,
AB/BC = tan 45º = 1
So, BC = AB = X
In ΔABD,
AB/BC = tan 45º = 1
So, BC = AB = X

In ΔABD,
AB/BD = tan 30º
X/(30 + X) = 1/ √3
30 + X = √3X
X = 30/ (√3 - 1) = 30/ (1.732 - 1) = 30/ 0.732
Thus, X = 40.98 m

2. Find the height of a tree when it is found that on walking away from it 20 m, in a horizontal line through its base, the elevation of its top changes from 60º to 30º.

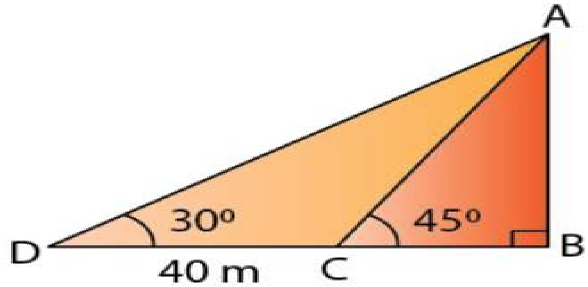
Solution:



Let's assume AB to be the height of the tree, h m.
Let the two points be C and D be such that CD = 20 m, ∠ADB = 30º and ∠ACB = 60º
In ΔABC,
AB/BC = tan 60º = √3
BC = AB/ √3 = h/ √3 ….. (i)
In ΔABD,
AB/BD = tan 30º
h/ (20 + BC) = 1/ √3
√3 h = 20 + BC
√3 h = 20 + h/ √3 …… [From (i)]
h (√3 – 1/√3) = 20
h = 20/ (√3 – 1/√3) = 20/ 1.154 = 17.32 m
Therefore, the height of the tree is 17.32 m.

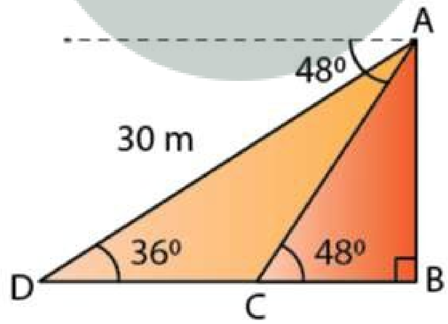
3. Find the height of a building, when it is found that on walking towards it 40 m in a horizontal

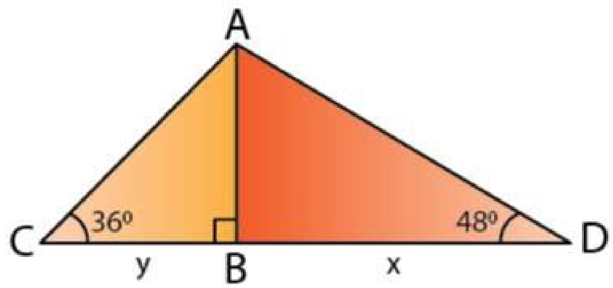
line through its base the angular elevation of its top changes from 30° to 45°.
Solution:



Let's assume AB to be the building of height h m.
Let the two points be C and D be such that CD = 40 m, ∠ADB = 30° and ∠ACB = 45°
In ΔABC,
AB/BC = tan 45° = 1
BC = AB = h
And, in ΔABD,
AB/ BD = tan 30°
h/ (40 + h) = 1/√3
√3h = 40 + h
h = 40/ (√3 - 1) = 40/ 0.732 = 54.64 m
Therefore, the height of the building is 54.64 m.

4. From the top of a light house 100 m high, the angles of depression of two ships are observed as 48° and 36° respectively. Find the distance between the two ships (in the nearest metre) if:
(i) the ships are on the same side of the light house.
(ii) the ships are on the opposite sides of the light house.
Solution:



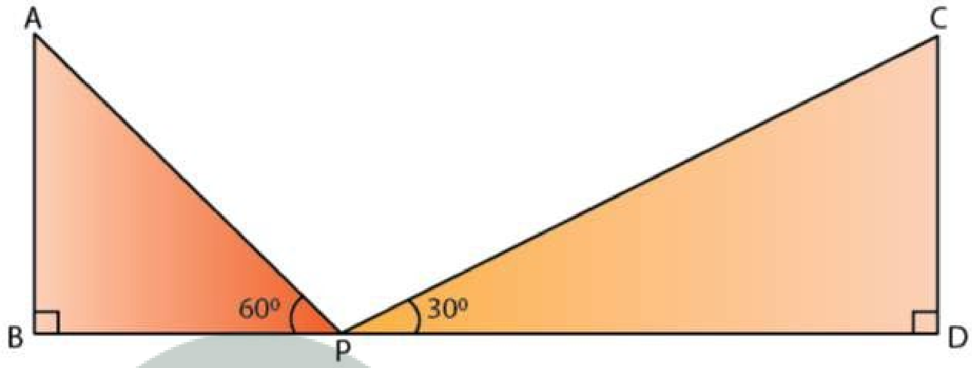


Let's consider AB to be the lighthouse.
And, let the two ships be C and D such that ∠ADB = 36° and ∠ACB = 48°
In ΔABC,
AB/BC = tan 48°
BC = 100/ 1.1106 = 90.04 m

In ΔABD,
AB/BD = tan 36°
BD = 100/ 0.7265 = 137.64 m
Now,

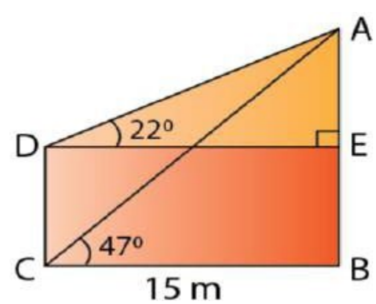
(i) If the ships are on the same side of the light house,
Then, the distance between the two ships = BD - BC = 48 m
(ii) If the ships are on the opposite sides of the light house,
Then, the distance between the two ships = BD + BC = 228 m

5. Two pillars of equal heights stand on either side of a roadway, which is 150 m wide. At a point in the roadway between the pillars the elevations of the tops of the pillars are 60° and 30°; find the height of the pillars and the position of the point.
Solution:



Let AB and CD be the two towers of height h m each.
And, let P be a point in the roadway BD such that BD = 150 m, ∠APB = 60° and ∠CPD = 30°
In ΔABP,
AB/BP = tan 60°
BP = h/ tan 60°
BP = h/ √3
In ΔCDP,
CD/DP = tan 30°
PD = √3 h
Now, 150 = BP + PD
150 = √3h + h/√3
h = 150/(√3 + 1/√3) = 150/ 2.309
h = 64.95 m
Thus, the height of the pillars are 64.95 m each.
Now,
The point is BP/ √3 from the first pillar.
Which is a distance of 64.95/ √3 from the first pillar.
Thus, the position of the point is 37.5 m from the first pillar.

6. From the figure, given below, calculate the length of CD.
Solution:

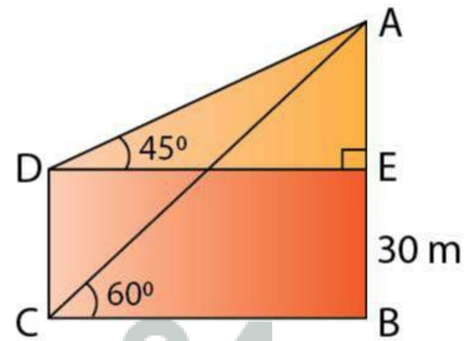


In ΔAED,
AE/ DE = tan 22°
AE = DE tan 22° = 15 x 0.404 = 6.06 m
In ΔABC,
AB/BC = tan 47°

AB = BC tan 47° = 15 x 1.072 = 16.09 m
Thus,
CD = BE = AB – AE = 10.03 m

7. The angle of elevation of the top of a tower is observed to be 60°. At a point, 30 m vertically above the first point of observation, the elevation is found to be 45°. Find:
(i) the height of the tower,
(ii) its horizontal distance from the points of observation.
Solution:

Let's consider AB to be the tower of height h m.
And let the two points be C and D be such that CD = 30 m, ∠ADE = 45° and ∠ACB = 60°



(i) In ΔADE,
AE/DE = tan 45° = 1
AE = DE
In ΔABC,
AB/BC = tan 60° = √3
AE + 30 = √3 BC
BC + 30 = √3 BC [Since, AE = DE = BC]
BC = 30/(√3 -1) = 30/ 0.732 = 40.98 m
Thus,
AB = 30 + 40.98 = 70.98 m
Thus, the height of the tower is 70.98 m

(ii) The horizontal distance from the points of observation is BC = 40.98 m