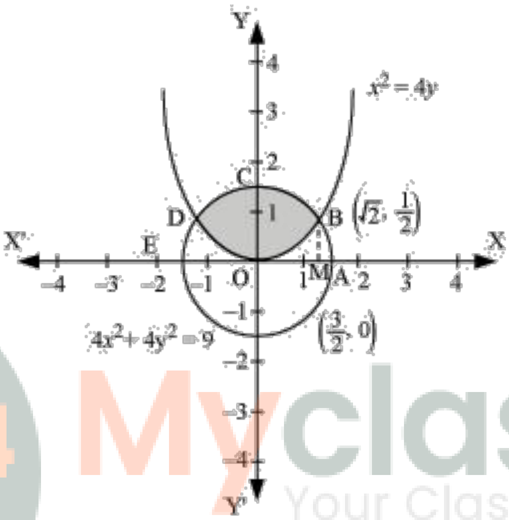
NCERT Solutions for Class-XII Maths

Chapter-8.2

NCERT Maths Class 12

1. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

1. The required area is represented by the shaded area OBCDO.



Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$, we obtain the point of intersection as $B\left(\sqrt{2}, \frac{1}{2}\right)$ and $D\left(-\sqrt{2}, \frac{1}{2}\right)$.

It can be observed that the required area is symmetrical about y-axis.

$\therefore$ Area OBCDO = 2 × Area OBCO

We draw BM perpendicular to OA.

Therefore, the coordinates of M are $\left(\sqrt{2}, 0\right)$.

Therefore, Area OBCO = Area OMBCO – Area OMBO

$$= \int_0^{\sqrt{2}} \sqrt{\frac{(9-4x^2)}{4}}\,dx - \int_0^{\sqrt{2}} \sqrt{\frac{x^2}{4}}\,dx$$

$$= \frac{1}{2}\int_0^{\sqrt{2}} \sqrt{9-4x^2}\,dx - \frac{1}{4}\int_0^{\sqrt{2}} x^2\,dx$$

$$= \frac{1}{4}\left[x\sqrt{9-4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}} - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$$

$$= \frac{1}{4}\left[\sqrt{2}\sqrt{9-8} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right] - \frac{1}{12}\left(\sqrt{2}\right)^3$$

$$= \frac{\sqrt{2}}{4} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6}$$

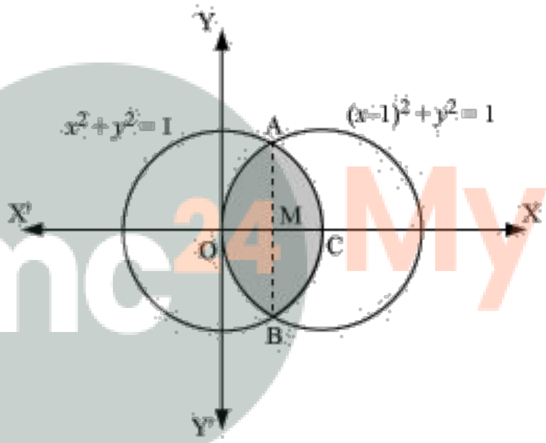
$$= \frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right)$$

Therefore, the required area OBCDO is

$$\left(2\times\frac{1}{2}\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]\right) = \left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]\text{units}$$

2. Find the area bounded by curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.



2.

It is given that area of circle, $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

On solving the above two equations, we get the point of intersection

$A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ *and* $B\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

We can see that the required area is symmetrical about x axis.

Thus, Area OBCAO = 2 × Area OCAO

Let us draw AM perpendicular to OC.

The coordinates of M are $(\frac{1}{2}, 0)$.

Then, Area OCAO = Area OMAO + Area MCAM

$$= \int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}dx + \int_{\frac{1}{2}}^{1}\sqrt{1-x^2}dx$$

$$= \left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_0^{\frac{1}{2}}$$

$$= \left[-\frac{1}{4}\sqrt{1-\left(-\frac{1}{2}\right)^2} + \frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2}\sin^{-1}(-1)\right] + \left[\frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\sqrt{1-\left(\frac{1}{2}\right)^2} - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= \left[-\frac{\sqrt{3}}{8} + \frac{1}{2}\left(-\frac{\pi}{6}\right) - \frac{1}{2}\left(-\frac{\pi}{2}\right)\right] + \left[\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} + \frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12}\right]$$

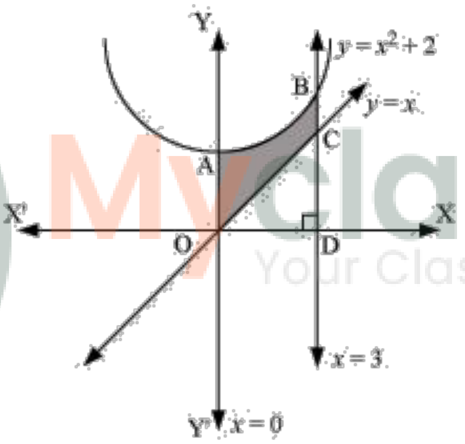
$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2}\right]$$

$$= \left[\frac{2\pi}{12} - \frac{\sqrt{3}}{4}\right]$$

Therefore, the required area OBCAO $= 2 \times \left[\frac{2\pi}{12} - \frac{\sqrt{3}}{4}\right] == \left[\frac{2\pi}{6} - \frac{\sqrt{3}}{2}\right] units.$

3. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

3. The area bounded by the curves, $y = x^2 + 2$, $y = x$, $x = 0$, and $x = 3$, is represented by the shaded area OCBAO as



Then, Area OCBAO = Area ODBAO – Area ODCO

$$= \int_0^3 \left(x^2 + 2\right)dx - \int_0^3 x dx$$

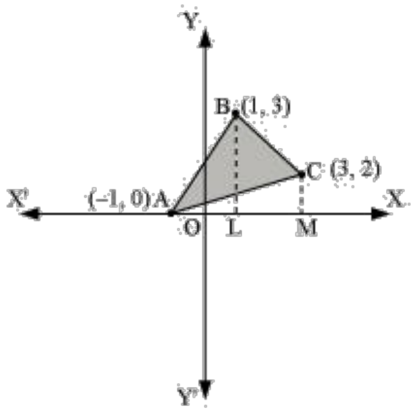
$$= \left[\frac{x^3}{3} + 2x\right]_0^3 - \left[\frac{x^2}{2}\right]_0^3$$

$$= [9 + 6] - \left[\frac{9}{2}\right]$$

$$= 15 - \frac{9}{2}$$

$$= \frac{21}{2} \text{ units}$$

4. Using integration finds the area of the region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).



4.

BL and CM are drawn perpendicular to x – axis.
We can see that from the figure that,
Area(ΔACB) = Area(ALBA) + Area(BLMCA) - Area(AMCA) … (1)
Now, equation of line segment AB is

$y - 0 = \frac{3-0}{1+1}(x+1)$

$\Rightarrow y = \frac{3}{2}(x+1)$

Thus, Area(ALBA) = $\int_{-1}^{1} \frac{3}{2}(x+1)dx = \frac{3}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{1}$

$= \frac{3}{2}\left[\frac{1}{2} + 1 - \frac{1}{2} + 1\right] = 3\ units$

Now, equation of line segment BC is

$y - 3 = \frac{2-3}{3-1}(x-1)$

$\Rightarrow y = \frac{1}{2}(-x+7)$

Thus, Area (BLMCB) = $\int_{1}^{3} \frac{1}{2}(-x+7)dx = \frac{1}{2}\left[-\frac{x^2}{2} + 7x\right]_{1}^{3}$

$= \frac{1}{2}\left[-\frac{9}{2} + 21 + \frac{1}{2} - 7\right] = 5\ units$

Now, equation of line segment AC is

$y - 0 = \frac{2-0}{3+1}(x+1)$

$\Rightarrow y = \frac{1}{2}(x+1)$

Thus, Area (AMCA) = $\int_{-1}^{3} \frac{1}{2}(x+1)dx = \frac{1}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{3}$

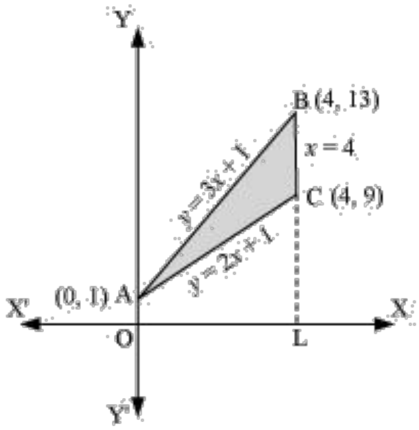
$= \frac{1}{2}\left[\frac{9}{2} + 3 - \frac{1}{2} + 1\right] = 4\ units$

Now putting all these values in equation (1), we get,
Area(ΔABC) = (3 + 5 – 4) = 4 units.

5. Using integration find the area of the triangular region whose sides have the equations y = 2x +1, y = 3x + 1 and x = 4.

5. The equations of sides of the triangle are $y = 2x + 1$, $y = 3x + 1$, and $x = 4$.

On solving these equations, we obtain the vertices of triangle as A(0, 1), B(4, 13), and C (4, 9).



It can be observed that,

Area (ΔACB) = Area (OLBAO) –Area (OLCAO)

$= \int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx$

$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left[\frac{2x^2}{2} + x\right]_0^4$

$= (24 + 4) - (16 + 4)$

$= 28 - 20$

$= 8$ units

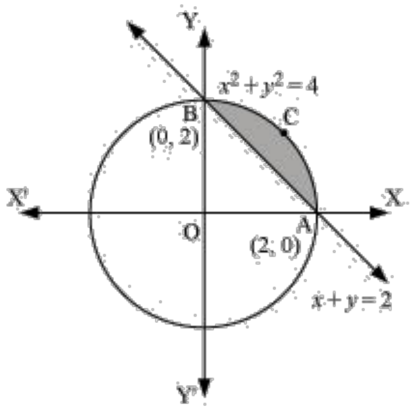
6. Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

(a) $2(\pi - 2)$ (b) $\pi - 2$

(c) $2\pi - 1$ (d) $2(\pi + 2)$

6. The correct option is (B).

The smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is shown by shaded region.



We can see that

Area ACBA = Area (OACBO) - Area (ΔOAB)

$= \int_0^2 \sqrt{4-x^2}dx - \int_0^2 (2-x)dx$

$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$

$= \left[2.\frac{\pi}{2}\right] - [4-2]$

$= (\pi - 2)$ units

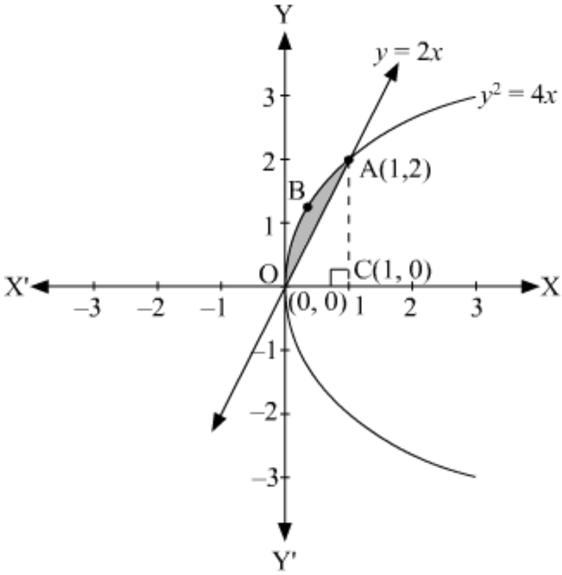
7. Area lying between the curve $y^2 = 4x$ and $y = 2x$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

7. The correct option is (B).

The area lying between the curve, $y^2 = 4x$ and $y = 2x$, is represented by the shaded area OBAO as



The points of intersection of these curves are O (0, 0) and A (1, 2).
We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).
∴ Area OBAO = Area (ΔOCA) – Area (OCABO)

$$= \int_0^1 2x\,dx - \int_0^1 2\sqrt{x}\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$$

$$= \left|1 - \frac{4}{3}\right|$$

$$= \left|-\frac{1}{3}\right|$$

$$= \frac{1}{3}\text{units}$$

Thus, the correct answer is B.