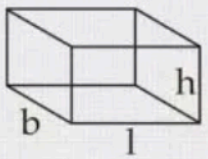
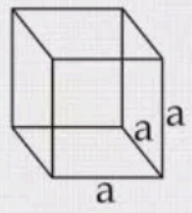

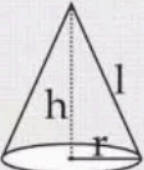
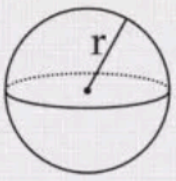

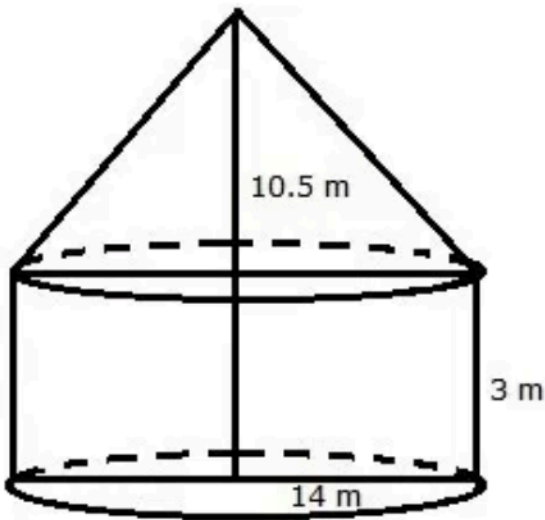


Volume & Surface Areas Of Solids

Exercise 19A

Name of the solid	Figure	Volume	Lateral/Curved Surface Area	Total Surface Area
Cuboid		lbh	$2lh + 2bh$ or $2h(l+b)$	$2lh+2bh+2lb$ or $2(lh+bh+lb)$
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi r h$	$2\pi r h + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3} \pi r^2 h$	$\pi r l$	$\pi r l + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3} \pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3} \pi r^3$	$2\pi r^2$	$2\pi r^2 + \pi r^2$ or $3\pi r^2$

Question 1:



Radius of the cylinder = 14 m

And its height = 3 m

Radius of cone = 14 m

And its height = 10.5 m

Let l be the slant height

$$\therefore l^2 = (14)^2 + (10.5)^2$$

$$l^2 = (196 + 110.25) \text{ m}^2$$

$$l^2 = 306.25 \text{ m}^2$$

$$l = \sqrt{306.25} \text{ m}$$

$$= 17.5 \text{ m}$$



Curved surface area of tent

= (curved area of cylinder + curved surface area of cone)

$$= 2\pi rh + \pi r l$$

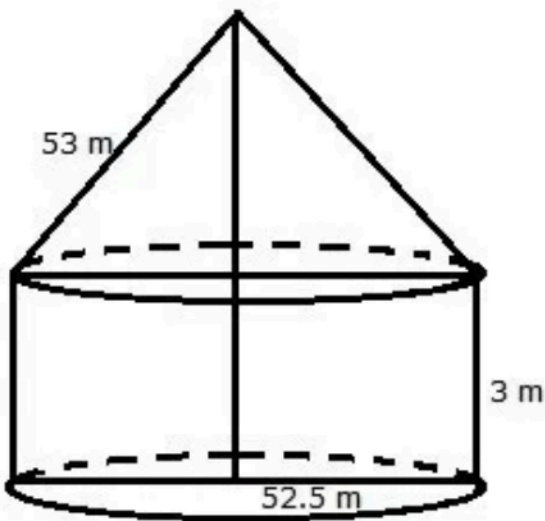
$$= \left[\left(2 \times \frac{22}{7} \times 14 \times 3 \right) + \left(\frac{22}{7} \times 14 \times 17.5 \right) \right] \text{ m}^2$$

$$= (264 + 770) \text{ m}^2 = 1034 \text{ m}^2$$

Hence, the curved surface area of the tent = 1034

Cost of canvas = Rs. (1034×80) = Rs. 82720

Question 2:



For the cylindrical portion, we have radius = 52.5 m and height = 3 m

For the conical portion, we have radius = 52.5 m

And slant height = 53 m

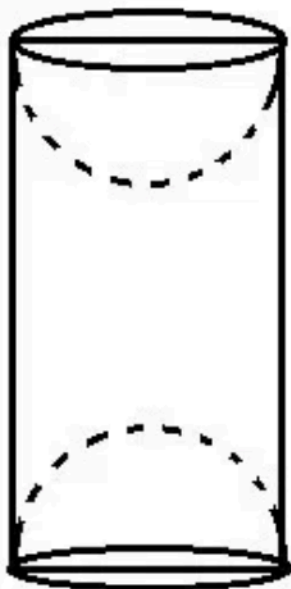
Area of canvas = $2rh + rl = r(2h + l)$

$$= \left[\frac{22}{7} \times 52.5 \times (2 \times 3 + 53) \right] m^2$$

$$= \left(22 \times \frac{15}{2} \times 59 \right) m^2 = 9735 m^2$$



Question 3:



Height of cylinder = 20 cm

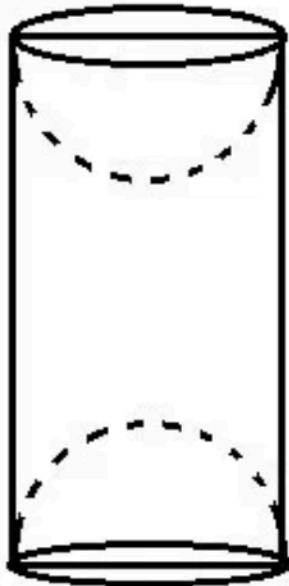
And diameter = 7 cm and then radius = 3.5 cm

Total surface area of article

= (lateral surface of cylinder with $r = 3.5$ cm and $h = 20$ cm)

$$\begin{aligned}
 &= [2\pi rh + 2 \times (2\pi r^2)] \text{ sq units} \\
 &= \left[\left(2 \times \frac{22}{7} \times \frac{7}{2} \times 20 \right) + \left(4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \right) \right] \text{ cm}^2 \\
 &= (440 + 154) \text{ cm}^2 = 594 \text{ cm}^2
 \end{aligned}$$

Question 4:



Radius of wooden cylinder = 4.2 cm

Height of wooden cylinder = 12 cm

Lateral surface area

$$\begin{aligned}
 &= 2\pi rh \text{ sq cm} \\
 &= 2 \times \pi \times 4.2 \times 12 \text{ cm}^2 \\
 &= 100.8\pi \text{ cm}^2
 \end{aligned}$$

Radius of hemisphere = 4.2 cm

Surface area of two hemispheres

$$\begin{aligned}
 &= 2 \times 2\pi r^2 \text{ sq unit} \\
 &= 4\pi \times 4.2 \times 4.2 \text{ cm}^2 \\
 &= 70.70\pi \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Total surface area} &= (100.8 + 70.56) \pi \text{ cm}^2 \\
 &= 538.56 \text{ cm}^2 \\
 &= 171.36 \pi \\
 &= 171.36 \times \frac{22}{7} \text{ cm}^2 \\
 &= 538.56 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned} \text{Further, volume of cylinder} &= \pi r^2 h = 4.2 \times 4.2 \times 12 \pi \text{ cm}^2 \\ &= 211.68 \pi \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Volume of two hemispheres} &= 2 \times \frac{2}{3} \pi r^3 \text{ cu.units} \\ &= \frac{4}{3} \pi \times 4.2 \times 4.2 \times 4.2 \\ &= 98.784 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of wood left} &= (211.68 - 98.784) \pi \\ &= 112.896 \pi \text{ cm}^3 \\ &= 112.896 \times \frac{22}{7} \text{ cm}^3 \\ &= 354.816 \text{ cm}^3 \end{aligned}$$

Question 5:

Radius of cylinder = 2.5 m

Height of cylinder = 21 m

Slant height of cone = 8 m

Radius of cone = 2.5 m

Total surface area of the rocket = (curved surface area of cone + curved surface area of cylinder + area of base)

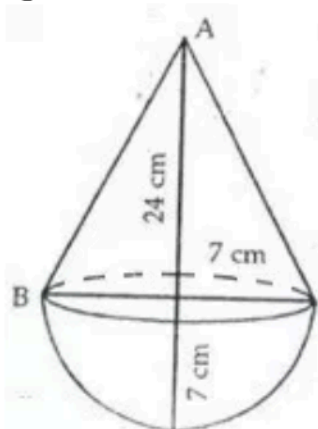
$$= (\pi r l + 2\pi r h + \pi r^2)$$

where $l = 8 \text{ m}$, $h = 21 \text{ m}$, $r = 2.5 \text{ m}$

$$= \left(\frac{22}{7} \times 2.5 \times 8 + 2 \times \frac{22}{7} \times 2.5 \times 21 + \frac{22}{7} \times 2.5 \times 2.5 \right) \text{m}^2$$

$$= (62.85 + 330 + 19.64) \text{m}^2 = 412.5 \text{m}^2$$

Question 6:



Height of cone = $h = 24 \text{ cm}$

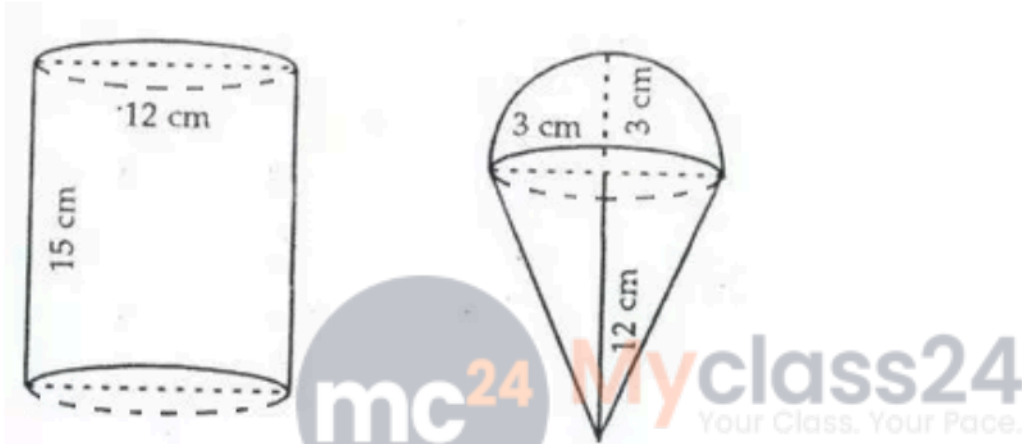
Its radius = 7 cm

$$\begin{aligned} \therefore \text{Slant height} &= \sqrt{(24)^2 + 7^2} \\ &= \sqrt{576 + 49} \\ &= \sqrt{625} = 25 \text{ cm} \end{aligned}$$

Total surface area of toy

$$\begin{aligned} &= (\pi r_1 + 2\pi r_1^2) \\ &= \pi (1 + 2r_1) \\ &= \frac{22}{7} \times 7 \times (25 + 14) \\ &= 22 \times 39 = 858 \text{ cm}^2 \end{aligned}$$

Question 7:



Height of cylindrical container $h_1 = 15$ cm

Diameter of cylindrical container = 12 cm

$$\text{Volume of container} = \pi r_1^2 h_1 = \pi \times 6 \times 6 \times 15 = 540\pi \text{ cm}^3$$

Height of cone $h_2 = 12$ cm

Diameter = 6 cm

Radius of $r_2 = 3$ cm

$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3} \pi r_2^2 h_2 = \frac{1}{3} \pi \times 3 \times 3 \times 12 \\ &= 36\pi \text{ cm}^3 \end{aligned}$$

Radius of hemisphere = 3 cm

$$\text{Volume of hemisphere} = \frac{2}{3} \pi r^3 = \frac{2}{3} \pi \times 3 \times 3 \times 3 = 36\pi$$

Volume of cone + volume of hemisphere

$$= 36\pi + 36\pi = 72\pi$$

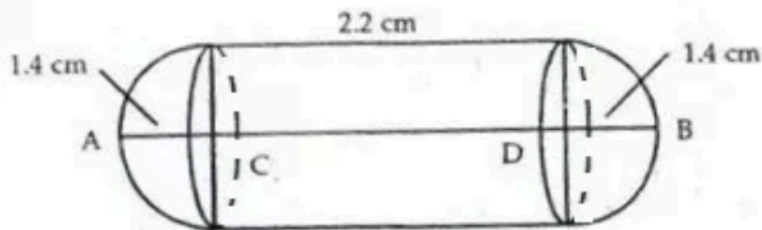
Number of cones

$$= \frac{\text{Volume of container}}{\text{Volume of cone} + \text{Volume of hemisphere}}$$

$$= \frac{540\pi}{54\pi} = 10$$

Number of cones that can be filled = 10

Question 8:



Diameter of cylindrical gulabjamun = 2.8 cm

Its radius = 1.4 cm

Total height of gulabjamun = AC + CD + DB = 5 cm

$$1.4 + CD + 1.4 = 5$$

$$2.8 + CD = 5$$

$$CD = 2.2 \text{ cm}$$

Height of cylindrical part $h = 2.2 \text{ cm}$

Volume of 1 gulabjamun = Volume of cylindrical part + Volume of two hemispherical parts

$$= \pi r^2 h + \frac{2}{3} \pi r^3 + \frac{2}{3} \pi r^3$$

$$= \pi r^2 h + \frac{4}{3} \pi r^3 = \pi r^2 \left(h + \frac{4}{3} r \right)$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times \left(2.2 + \frac{4}{3} \times 1.4 \right)$$

$$= 22 \times 0.2 \times 1.4 \times (2.2 + 1.87)$$

$$= 4.4 \times 1.4 \times 4.07 = 25.07 \text{ cm}^3$$

Volume of 45 gulabjamuns = $45 \times 25.07 \text{ cm}^3$

Quantity of syrup = 30% of volume of gulabjamuns

$$= 0.3 \times 45 \times 25.07 = 338.46 \text{ cm}^3$$

Question 9:

Diameter = 7cm, radius = 3.5 cm

Height of cone = 14.5 cm - 3.5 cm = 11 cm

$$l = \sqrt{\left(\frac{7}{2}\right)^2 + (11)^2} \text{ cm} = \sqrt{\frac{49}{4} + 121} \text{ cm} = \sqrt{\frac{533}{4}} \text{ cm}$$

$$l = \frac{23.08}{2} \text{ cm} = 11.54 \text{ cm}$$

$$\text{Volume of toy} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \left[\frac{1}{3}\pi r^2 (2r + h)\right]$$

where $r = \frac{7}{2}$ and $h = 11$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left(2 \times \frac{7}{2} + 11\right)\right] \text{ cm}^3$$

$$= (12.83 \times 18) \text{ cm}^3 = 230.94 \text{ cm}^3$$

$$\text{Total surface area of toy} = (2\pi r^2 + \pi r l) \text{ cm}^2 = \pi (2r + l) \text{ cm}^2$$

$$= \frac{22}{7} \times \frac{7}{2} \times \left(2 \times \frac{7}{2} + 11.54\right) \text{ cm}^2$$

$$= (11 \times 18.54) \text{ cm}^2 = 203.94 \text{ cm}^2$$

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Question 10:

Diameter of cylinder = 24 m

Radius of cylinder = $\frac{24}{2} = 12$ m

Height of the cylinder = 11 m

Height of cone = (16 - 11) m = 5 m

$$\text{Slant height of the cone } l = \sqrt{r^2 + h^2} = \sqrt{144 + 25} \text{ m} = 13 \text{ m}$$

Area of canvas required = (curved surface area of the cylindrical part) + (curved surface area of the conical part)

$$= (2\pi r h + \pi r l) \text{ m}^2 = \pi (2h + l) \text{ m}^2$$

$$= \left[\frac{22}{7} \times 12 \times (2 \times 11 + 13)\right] \text{ m}^2$$

$$= \left(\frac{22}{7} \times 12 \times 35\right) \text{ m}^2 = 1320 \text{ m}^2$$

Question 11:

Radius of hemisphere = 10.5 cm

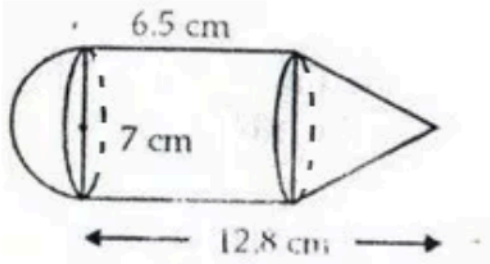
Height of cylinder = (14.5 - 10.5) cm = 4 cm

Radius of cylinder = 10.5 cm

Capacity = Volume of cylinder + Volume of hemisphere

$$\begin{aligned}
 &= \left(\pi r^2 h + \frac{2}{3} \pi r^3 \right) \text{cm}^3 = \pi r^2 \left(h + \frac{2}{3} r \right) \text{cm}^3 \\
 &= \left[\frac{22}{7} \times 10.5 \times 10.5 \times \left(4 + \frac{2}{3} \times 10.5 \right) \right] \text{cm}^3 \\
 &= (346.5 \times 11) \text{cm}^2 = 3811.5 \text{cm}^2
 \end{aligned}$$

Question 12:



Height of cylinder = 6.5 cm

Height of cone = $h_2 = (12.8 - 6.5) \text{ cm} = 6.3 \text{ cm}$

Radius of cylinder = radius of cone

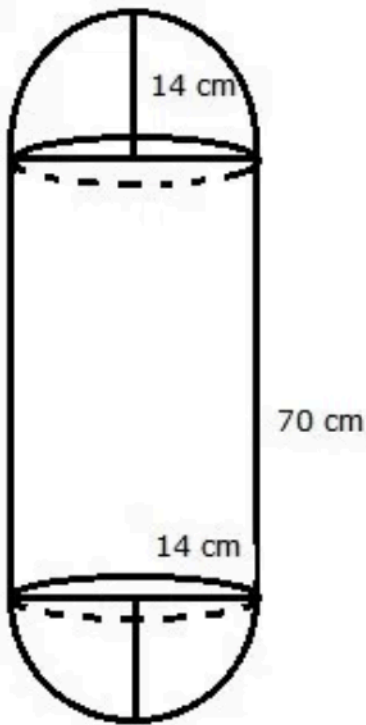
= radius of hemisphere

= $\frac{7}{2} \text{ cm}$

Volume of solid = Volume of cylinder + Volume of cone + Volume of hemisphere

$$\begin{aligned}
 &= \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2 + \frac{2}{3} \pi r^3 = \pi r^2 \left(h_1 + \frac{1}{3} h_2 + \frac{2}{3} r \right) \\
 &= \left[\frac{22}{7} \times 3.5 \times 3.5 \times \left(6.5 + 6.3 \times \frac{1}{3} + \frac{2}{3} \times 3.5 \right) \right] \\
 &= [(38.5) \times (6.5 + 2.1 + 2.33)] \text{cm}^3 \\
 &= (38.5 \times 10.93) \text{cm}^3 = 420.80 \text{cm}^3
 \end{aligned}$$

Question 13:



Radius of each hemispherical end = $\frac{28}{2} = 14$ cm

Height of each hemispherical part = Its Radius

Height of cylindrical part = $(98 - 2 \times 14) = 70$ cm

Area of surface to be polished = $2(\text{curved surface area of hemisphere}) + (\text{curved surface area of cylinder})$

$$= [2(2\pi r^2) + 2\pi rh] \text{sq unit}$$

$$= 2\pi (2r + h) \text{cm}^2$$

$$= 2 \times \frac{22}{7} \times 14 \times [2 \times 14 + 70] \text{cm}^2$$

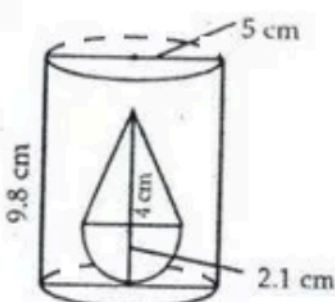
$$= (88 \times 98) = 8624 \text{cm}^2$$

Cost of polishing the surface of the solid

$$= \text{Rs. } (0.15 \times 8624)$$

$$= \text{Rs. } 1293.60$$

Question 14:



Radius of cylinder $r_1 = 5$ cm

And height of cylinder $h_1 = 9.8$ cm

Radius of cone $r = 2.1$ cm

And height of cone $h_2 = 4$ cm

Volume of water left in tub = (volume of cylindrical tub – volume of solid)

$$= \left(\pi r^2 h_1 - \frac{2}{3} \pi r^3 - \frac{1}{3} \pi r^2 h_2 \right)$$

$$= \left(\frac{22}{7} \times 5 \times 5 \times 9.8 - \frac{2}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 - \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4 \right)$$

$$= [(770 - 19.404) - 18.48] \text{ cm}^3$$

$$= 732.116 \text{ cm}^3$$

Question 15:

(i) Radius of cylinder = 6 cm

Height of cylinder = 8 cm



Volume of cylinder

$$\Rightarrow \pi r^2 \times 10800 = 972\pi$$

$$r^2 = \frac{972\pi}{10800\pi} = 0.09 \text{ cm}^2$$

$$r = \sqrt{0.09} \text{ cm} = 0.3$$

Volume of cone removed

$$= \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8 \text{ cm}^3$$

$$= 96 \pi \text{ cm}^3$$

(ii) Surface area of cylinder = $2\pi r h = 2\pi \times 6 \times 8 \text{ cm}^2 = 96 \pi \text{ cm}^2$

$$\begin{aligned} \text{Slant height of cone} &= \sqrt{6^2 + 8^2} = \sqrt{36 + 64} \text{ cm} \\ &= \sqrt{100} \text{ cm} = 10 \text{ cm} \end{aligned}$$

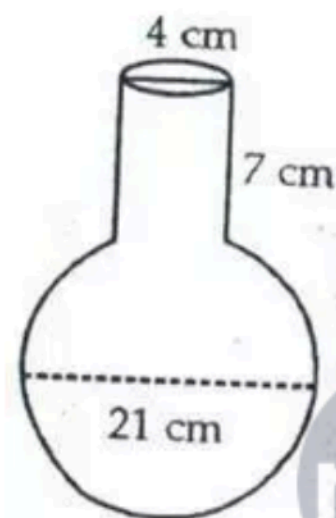
$$\text{Curved surface area of cone} = \pi r l = \pi \times 6 \times 10 = 60 \pi$$

$$\text{Area of base of cylinder} = \pi r^2 = \pi \times 6 \times 6 = 36 \pi$$

Total surface area of remaining solid

$$\begin{aligned} &= (96\pi + 60\pi + 36\pi) \text{ cm}^2 \\ &= 192\pi \text{ cm}^2 = 602.88 \text{ cm}^2 \end{aligned}$$

Question 16:



Diameter of spherical part of vessel = 21 cm

$$\text{Its radius} = \frac{21}{2} \text{ cm}$$

$$\text{Its volume} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$

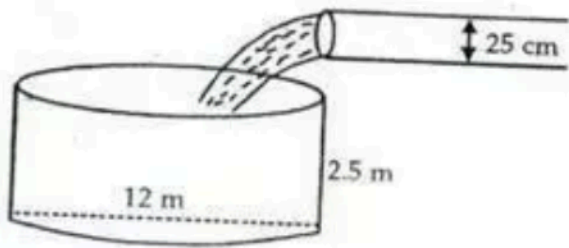
$$= 11 \times 21 \times 21 \text{ cm}^3 = 4851 \text{ cm}^3$$

Volume of cylindrical part of vessel

$$= \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 7 \text{ cm}^3$$

$$= 88 \text{ cm}^3$$

$$\therefore \text{Volume of whole vessel} = (4851 + 88) \text{ cm}^3 = 4939 \text{ cm}^3$$

Question 17:

Height of cylindrical tank = 2.5 m
 Its diameter = 12 m, Radius = 6 m

$$\text{Volume of tank} = \pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 2.5 \text{ m}^3 = \frac{1980}{7} \text{ m}^3$$

Water is flowing at the rate of 3.6 km/ hr = 3600 m/hr

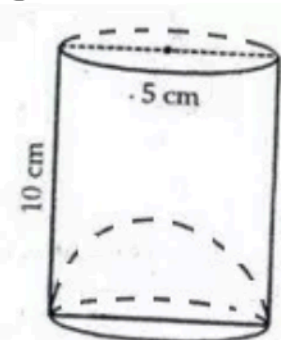
Diameter of pipe = 25 cm, radius = 0.125 m

Volume of water flowing per hour

$$\begin{aligned} &= \frac{22}{7} \times 0.125 \times 0.125 \times 3600 \text{ m}^3 \\ &= \frac{22 \times 3600}{7 \times 8 \times 8} \text{ m}^3 = \frac{2475}{14} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Time taken to fill the tank} &= \frac{1980}{7} \div \frac{2475}{14} \text{ hr} \\ &= \frac{1980}{7} \times \frac{14}{2475} \text{ hr} = \frac{792}{495} \text{ hr} \\ &= 1.36 \text{ hr} = 1 \text{ hr } 36 \text{ min.} \end{aligned}$$

$$\text{Water charges} = \text{Rs. } \frac{1980}{7} \times 0.07 = \text{Rs. } 19.80$$

Question 18:

Diameter of cylinder = 5 cm

Radius = 2.5 cm

Height of cylinder = 10 cm

$$\text{Volume of cylinder} = \pi r^2 h \text{ cu. units} = 3.14 \times 2.5 \times 2.5 \times 10 \text{ cm}^3 = 196.25 \text{ cm}^3$$

Apparent capacity of glass = 196.25

Radius of hemisphere = 2.5 cm

Volume of hemisphere

$$= \frac{2}{3} \pi r^3$$

$$= \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5 \text{ cm}^3$$

$$= 32.708 \text{ cm}^3$$

$$\text{Actual capacity of glass} = (196.25 - 32.608) \text{ cm}^3 = 163.54 \text{ cm}^3$$

Exercise 19B

https://www.youtube.com/watch?v=6KpStN_0mjE

Question 1:

Radius of the cone = 12 cm and its height = 24 cm

$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3} \pi r^2 h = \left(\frac{1}{3}\right) \times 12 \times 12 \times 24 \text{ cm}^3 \\ &= (48 \times 24) \pi \text{ cm}^3 \\ &= (48 \times 24) \pi \text{ cm}^3 \end{aligned}$$

$$\text{Volume of each ball} = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi \times 3 \times 3 \times 3 = (36\pi) \text{ cm}^3$$

$$\text{Number of balls formed} = \frac{\text{Volume of solid cone}}{\text{Volume of each ball}}$$

$$= \frac{(48 \times 24\pi)}{36\pi} = 32$$

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Question 2:

Internal radius = 3 cm and external radius = 5 cm

$$\begin{aligned} \text{Volume of material in the shell} &= \frac{2}{3} \pi \times [(5)^3 - (3)^3] \text{ cm}^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times 98 = \frac{616}{3} \text{ cm}^3 \end{aligned}$$

Radius of the cone = 7 cm

Let height of cone be h cm

$$\text{Volume of cone} = \left(\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h\right) \text{ cm}^3 = \frac{154h}{3} \text{ cm}^3$$

$$\therefore \frac{154h}{3} = \frac{616}{3}$$

$$\Rightarrow h = \frac{616}{154} = 4 \text{ cm}$$

Hence, height of the cone = 4 cm

Question 3:

Inner radius of the bowl = 15 cm

Volume of liquid in it =

$$\frac{2}{3} \pi r^3 = \left(\frac{2}{3} \pi \times (15)^3 \right) \text{cm}^3$$

Radius of each cylindrical bottle = 2.5 cm and its height = 6 cm

Volume of each cylindrical bottle

$$= \pi r^2 h = \left(\pi \times \left(\frac{5}{2} \right)^2 \times 6 \right) \text{cm}^2$$

$$= \left(\frac{25}{4} \times 6\pi \right) = \left(\frac{75\pi}{2} \right) \text{cm}^3$$

Required number of bottles = $\frac{\text{Volume of liquid}}{\text{Volume of each cylindrical bottle}}$

$$= \frac{\frac{2}{3} \times \pi \times 15 \times 15 \times 15}{\frac{75}{2} \times \pi} = 60$$

Hence, bottles required = 60

Question 4:

Radius of the sphere = $\frac{21}{2}$ cm

$$\text{Volume of the sphere} = \left(\frac{4}{3} \pi r^3 \right) = \left[\frac{4}{3} \pi \times \left(\frac{21}{2} \right)^3 \right] \text{cm}^3$$

Radius of cone = $\frac{7}{4}$ cm and height 3 cm

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h = \left(\frac{1}{3} \times \pi \times \left(\frac{7}{4} \right)^2 \times 3 \right) \text{cm}^3$$

Let the number of cones formed be n, then

$$n \times \frac{1}{3} \pi \times \left(\frac{7}{4} \right)^2 \times 3 = \frac{4}{3} \pi \times \left(\frac{21}{2} \right)^3$$

$$n = \frac{4}{3} \pi \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \times \frac{3}{\pi} \times \frac{4}{7} \times \frac{4}{7} \times \frac{1}{3}$$

$$n = 504$$

Hence, number of cones formed = 504

Question 5:

Radius of the cannon ball = 14 cm

$$\text{Volume of cannon ball} = \frac{4}{3} \pi r^3 = \left[\frac{4}{3} \pi \times (14)^3 \right] \text{cm}^3$$

Radius of the cone = $\frac{35}{2}$ cm

Let the height of cone be h cm

$$\text{Volume of cone} = \left[\frac{1}{3} \pi \times \left(\frac{35}{2} \right)^2 \times h \right] \text{cm}^3$$

$$\therefore \frac{4}{3} \pi \times (14)^3 = \frac{1}{3} \pi \times \left(\frac{35}{2}\right)^2 \times h$$

$$h = \frac{4}{3} \pi \times 14 \times 14 \times 14 \times \frac{3}{\pi} \times \frac{2}{35} \times \frac{2}{35}$$

$$= 35.84 \text{ cm}$$

Hence, height of the cone = 35.84 cm

Question 6:

Let the radius of the third ball be r cm, then,

Volume of third ball = Volume of spherical ball – volume of 2 small balls

$$\text{Volume of third ball} = \left[\frac{4}{3} \pi (3)^3 - \left\{ \frac{4}{3} \pi \left(\frac{3}{2}\right)^3 + \frac{4}{3} \pi (2)^3 \right\} \right]$$

$$= \left[36\pi - \left(\frac{9\pi}{2} + \frac{32\pi}{3} \right) \right] \text{cm}^3 = \frac{125\pi}{6} \text{cm}^3$$

$$\therefore \frac{4}{3} \pi r^3 = \frac{125\pi}{6}$$

$$r^3 = \frac{125\pi \times 3}{6 \times 4 \times \pi} = \frac{125}{8}$$

$$r = \left(\frac{5}{2}\right) \text{cm} = 2.5 \text{ cm}$$

Question 7:

External radius of shell = 12 cm and internal radius = 9 cm

$$\text{Volume of lead in the shell} = \frac{4}{3} \pi \left[(12)^3 - (9)^3 \right] \text{cm}^3$$

Let the radius of the cylinder be r cm

Its height = 37 cm

$$\text{Volume of cylinder} = \pi r^2 h = (\pi r^2 \times 37)$$

$$\therefore \frac{4}{3} \pi \left[(12)^3 - (9)^3 \right] = \pi r^2 \times 37$$

$$\frac{4}{3} \times \pi \times 999 = \pi r^2 \times 37$$

$$r^2 = \frac{4}{3} \times \pi \times 999 \times \frac{1}{37\pi} = 36 \text{ cm}^2$$

$$r = \sqrt{36} \text{ cm} = 6 \text{ cm}$$

Hence diameter of the base of the cylinder = 12 cm

Question 8:

Volume of hemisphere of radius 9 cm

$$= \left(\frac{2}{3} \times \pi \times 9 \times 9 \times 9 \right) \text{cm}^3$$

Volume of circular cone (height = 72 cm)

$$= \frac{1}{3} (\pi \times r^2 \times 72) \text{cm}$$

Volume of cone = Volume of hemisphere

$$\therefore \frac{1}{3} \times \pi r^2 \times 72 = \frac{2}{3} \pi \times 9 \times 9 \times 9$$

$$r^2 = \frac{2\pi}{3} \times 9 \times 9 \times 9 \times \frac{1}{24\pi} = 20.25$$

$$r = \sqrt{20.25} = 4.5 \text{ cm}$$

Hence radius of the base of the cone = 4.5 cm

Question 9:

Diameter of sphere = 21 cm

Hence, radius of sphere = $\frac{21}{2}$ cm

Volume of sphere = $\frac{4}{3} \pi r^3 = \left(\frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}\right)$

Volume of cube = $a^3 = (1 \times 1 \times 1)$

Let number of cubes formed be n

\therefore Volume of sphere = n \times Volume of cube

$$\therefore \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} = n \times 1$$

$$= (441 \times 11) = n$$

$$4851 = n$$

Hence, number of cubes is 4851.

Question 10:

Volume of sphere (when $r = 1$ cm) = $\frac{4}{3} \pi r^3 = \left(\frac{4}{3} \times 1 \times 1 \times 1\right) \pi \text{ cm}^3$

Volume of sphere (when $r = 8$ cm) = $\frac{4}{3} \pi r^3 = \left(\frac{4}{3} \times 8 \times 8 \times 8\right) \pi \text{ cm}^3$

Let the number of balls = n

$$n \times \left(\frac{4}{3} \times 1 \times 1 \times 1\right) \pi = \left(\frac{4}{3} \times 8 \times 8 \times 8\right) \pi$$

$$n = \frac{4 \times 8 \times 8 \times 8 \times 3}{3 \times 4} = 512$$

Question 11:

Radius of marbles = $\frac{\text{Diameter}}{2} = \frac{1.4}{2} \text{ cm}$

$$\begin{aligned} \text{Volume of marbles} &= \frac{4}{3} \pi r^3 \\ &= \left[\frac{4}{3} \times \pi \times \left(\frac{1.4}{2}\right) \times \left(\frac{1.4}{2}\right) \times \left(\frac{1.4}{2}\right) \right] \text{cm}^3 \end{aligned}$$

$$\text{Radius of beaker} = \left(\frac{7}{2}\right) \text{cm}$$

$$\begin{aligned} \text{Volume of rising water in beaker} \\ &= \pi r^2 h = \left(\pi \times \left(\frac{7}{2}\right)^2 \times \left(\frac{56}{10}\right) \right) \text{cm}^3 \end{aligned}$$

Let the number of marbles be n

$\therefore n \times \text{volume of marble} = \text{volume of rising water in beaker}$

$$n \times \left(\frac{4}{3} \pi \times \frac{1.4}{2} \times \frac{1.4}{2} \times \frac{1.4}{2} \right) = \pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10}$$

$$n = 150$$

Hence the number of marbles is 150

Question 12:

Radius of sphere = 3 cm

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3 = \left(\frac{4}{3} \times \pi \times 3 \times 3 \times 3 \right) \pi \text{ cm}^3 = 36\pi \text{ cm}^3$$

Radius of small sphere = $\frac{0.6}{2}$ cm = 0.3 cm

$$\text{Volume of small sphere} = \left(\frac{4}{3} \times \pi \times 0.3 \times 0.3 \times 0.3 \right) \pi \text{ cm}^3$$

$$= \left(\frac{4}{3} \times \pi \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \right) \text{cm}^3$$

$$= \left(\frac{4\pi}{3} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \right) \text{cm}^3$$

Let number of small balls be n

$$n \times \left(\frac{4\pi}{3} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \right) = \frac{4}{3} \pi \times 3 \times 3 \times 3$$

$$n = 1000$$

Hence, the number of small balls = 1000.

Question 13:

Diameter of sphere = 42 cm

Radius of sphere = $\frac{42}{2}$ cm = 21 cm

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3 = \left(\frac{4}{3} \times \pi \times 21 \times 21 \times 21 \right) \pi \text{ cm}^3$$

Diameter of cylindrical wire = 2.8 cm

Radius of cylindrical wire = $\frac{2.8}{2}$ cm = 1.4 cm

Volume of cylindrical wire = $\pi r^2 h = (\pi \times 1.4 \times 1.4 \times h) \text{ cm}^3 = (1.96\pi h) \text{ cm}^3$

Volume of cylindrical wire = volume of sphere

$$\therefore 1.96\pi h = \frac{4}{3} \times \pi \times 21 \times 21 \times 21$$

$$h = \left(\frac{4}{3} \times \pi \times 21 \times 21 \times 21 \times \frac{1}{1.96} \times \frac{1}{\pi} \right) \text{ cm}$$

$$h = 6300$$

$$h \left(\frac{6300}{100} \right) \text{ m} = 63 \text{ m}$$

Hence length of the wire 63 m.

Question 14:

Diameter of sphere = 6 cm

Radius of sphere = $\frac{6}{2} \text{ cm} = 3 \text{ cm}$

Volume of sphere = $\frac{4}{3} \pi r^3 = \left(\frac{4}{3} \times 3 \times 3 \times 3 \right) \pi \text{ cm}^3 = 36\pi \text{ cm}^3$

Radius of wire = $\frac{2}{2} \text{ mm} = 1 \text{ mm} = 0.1 \text{ cm}$

Volume of wire = $\pi r^2 l = (\pi \times 0.1 \times 0.1 \times l) \text{ cm}^2 = (0.01 \pi l) \text{ cm}^2$

$$36\pi = 0.01 \pi l$$

$$\therefore l = \frac{36}{0.01} = 3600$$

Length of wire = $\frac{3600}{100} \text{ m} = 36 \text{ m}$



Question 15:

Diameter of sphere = 18 cm

Radius of copper sphere = $\frac{3600}{100} \text{ m} = 36 \text{ m}$

Volume of sphere = $\left(\frac{4}{3} \times \pi \times r^3 \right) \text{ cm}^3$

$$= \left(\frac{4}{3} \pi \times 9 \times 9 \times 9 \right) \text{ cm}^3 = 972 \pi \text{ cm}^3$$

Length of wire = 108 m = 10800 cm

Let the radius of wire be $r \text{ cm}$

$$= \pi r^2 l \text{ cm}^3 = (\pi r^2 \times 10800) \text{ cm}^3$$

But the volume of wire = Volume of sphere

$$\Rightarrow \pi r^2 \times 10800 = 972\pi$$

$$r^2 = \frac{972\pi}{10800\pi} = 0.09 \text{ cm}^2$$

$$r = \sqrt{0.09} \text{ cm} = 0.3$$

Hence the diameter = $2r = (0.3 \times 2) \text{ cm} = 0.6 \text{ cm}$

Question 16:

The radii of three metallic spheres are 3 cm, 4 cm and 5 cm respectively.

$$\begin{aligned} \text{Sum of their volumes} &= \frac{4}{3} \pi (3^3 + 4^3 + 5^3) \text{ cm}^3 \\ &= \frac{4}{3} \pi (27 + 64 + 125) = \frac{4}{3} \pi \times 216 \end{aligned}$$

Let r be the radius of sphere whose volume is equal to the total volume of three spheres.

$$\frac{4}{3} \pi r^3 = \frac{4}{3} \pi \times 216$$

$$\Rightarrow r^3 = 216$$

$$\therefore r = 6 \text{ cm}$$

$$\therefore \text{Diameter} = 6 \times 2 = 12 \text{ cm}$$

Exercise 19C

Question 1:

Here $h = 42 \text{ cm}$, $R = 16 \text{ cm}$, and $r = 11 \text{ cm}$

$$\begin{aligned} \text{Capacity} &= \frac{1}{3} \pi h (R^2 + r^2 + Rr) \text{ cm}^3 \\ &= \frac{1}{3} \times \frac{22}{7} \times 42 [(16)^2 + (11)^2 + 16 \times 11] \text{ cm}^3 \\ &= (44 \times 553) \text{ cm}^3 = 24332 \text{ cm}^3 \end{aligned}$$

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Question 2:

Here $R = 33 \text{ cm}$, $r = 27 \text{ cm}$ and $l = 10 \text{ cm}$

$$\begin{aligned} \therefore h &= \sqrt{l^2 - (R^2 - r^2)} \text{ cm} = \sqrt{(10)^2 - (33 - 27)^2} \text{ cm} \\ &= \sqrt{(10)^2 - (6)^2} = \sqrt{64} \text{ cm} = 8 \text{ cm} \end{aligned}$$

Capacity of the frustum

$$\begin{aligned} &= \frac{1}{3} \pi h (R^2 + r^2 + Rr) \text{ cm}^3 \\ &= \frac{1}{3} \times \frac{22}{7} \times 8 [(33)^2 + (27)^2 + 33 \times 27] \text{ cm}^3 \\ &= (8.38 \times 2709) \text{ cm}^3 = 22701.4 \text{ cm}^3 \end{aligned}$$

Total surface area

$$\begin{aligned}
 &= [\pi R^2 + \pi r^2 + \pi l(R+r)] \text{cm}^2 \\
 &= \pi [R^2 + r^2 + l(R+r)] \text{cm}^2 \\
 &= \frac{22}{7} [(33)^2 + (27)^2 + 10 \times (33+27)] \text{cm}^2 \\
 &= \left(\frac{22}{7} \times 2418\right) \text{cm}^2 = 7599.43 \text{cm}^2
 \end{aligned}$$

Question 3:

Height = 15 cm, $R = \frac{56}{2}$ cm = 28 cm and $r = \frac{42}{2}$ cm = 21 cm
Capacity of the bucket =

$$\begin{aligned}
 &\frac{1}{3} \pi h (R^2 + r^2 + Rr) \text{cm}^3 \\
 &= \frac{1}{3} \times \frac{22}{7} \times 15 [(28)^2 + (21)^2 + 28 \times 21] \text{cm}^3 \\
 &= (15.71 \times 1831) \text{cm}^3 \\
 &= (28482.23) \text{cm}^3
 \end{aligned}$$

Quantity of water in bucket = 28.49 litres

Question 4:

$R = 20$ cm, $r = 8$ cm and $h = 16$ cm

$$\begin{aligned}
 \therefore l &= \sqrt{h^2 + (R-r)^2} = \sqrt{(16)^2 + (20-8)^2} \\
 &= \sqrt{256 + 144} \text{cm} = 20 \text{cm}
 \end{aligned}$$

$$\begin{aligned}
 \text{Total surface area of container} &= \pi l (R+r) + \pi r^2 \\
 &= [3.14 \times 20 \times (20+8) + 3.14 \times 8 \times 8] \text{cm}^2 \\
 &= (3.14 \times 20 \times 28 + 3.14 \times 8 \times 8) \text{cm}^2 \\
 &= (1758.4 + 200.96) \text{cm}^2 \\
 &= 1959.36 \text{cm}^2
 \end{aligned}$$

Cost of metal sheet used = Rs. $(1959.36 \times \frac{15}{100})$ = Rs. 293.90

Question 5:

$R = 15$ cm, $r = 5$ cm and $h = 24$ cm

$$\begin{aligned}
 \therefore l &= \sqrt{h^2 + (R-r)^2} = \sqrt{(24)^2 + (10)^2} \text{cm} \\
 &= \sqrt{576 + 100} \text{cm} = \sqrt{676} \text{cm} = 26 \text{cm}
 \end{aligned}$$

(i) Volume of bucket =

$$\frac{1}{3} \pi h (R^2 + r^2 + Rr)$$

$$= \frac{1}{3} \times 3.14 \times 24 \times [(15)^2 + (5)^2 + 15 \times 5]$$

$$= (25.12 \times 325) \text{ cm}^3$$

$$= 8164 \text{ cm}^3 = 8.164 \text{ litres}$$

Cost of milk = Rs. $(8.164 \times 20) = \text{Rs. } 163.28$

(ii) Total surface area of the bucket

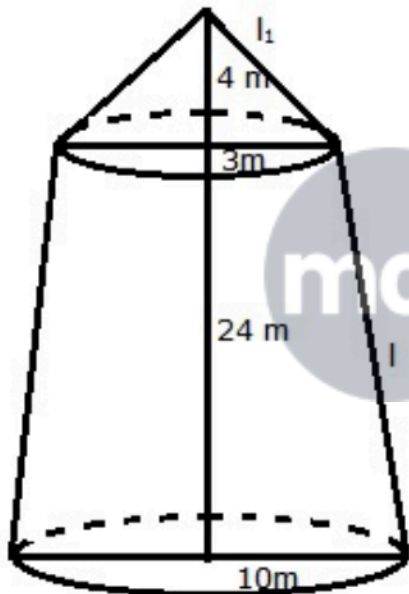
$$= \pi(R + r)l + \pi r^2$$

$$= (3.14 \times 26 \times 20 + 3.14 \times 5 \times 5) \text{ cm}^2$$

$$= 1711.3 \text{ cm}^2$$

Cost of sheet = $(1711.3 \times \frac{10}{100}) = \text{Rs. } 171.13$

Question 6:



$R = 10\text{cm}$, $r = 3\text{ m}$ and $h = 24\text{ m}$

Let l be the slant height of the frustum, then

$$\begin{aligned}
 l &= \sqrt{h^2 + (R - r)^2} \\
 &= \sqrt{(24)^2 + (10 - 3)^2} \\
 &= \sqrt{(24)^2 + (7)^2} \\
 &= \sqrt{576 + 49} \\
 &= \sqrt{625} \text{ m} = 25 \text{ m}
 \end{aligned}$$

Let l_1 be the slant height of conical part

$$r = 3 \text{ m}$$

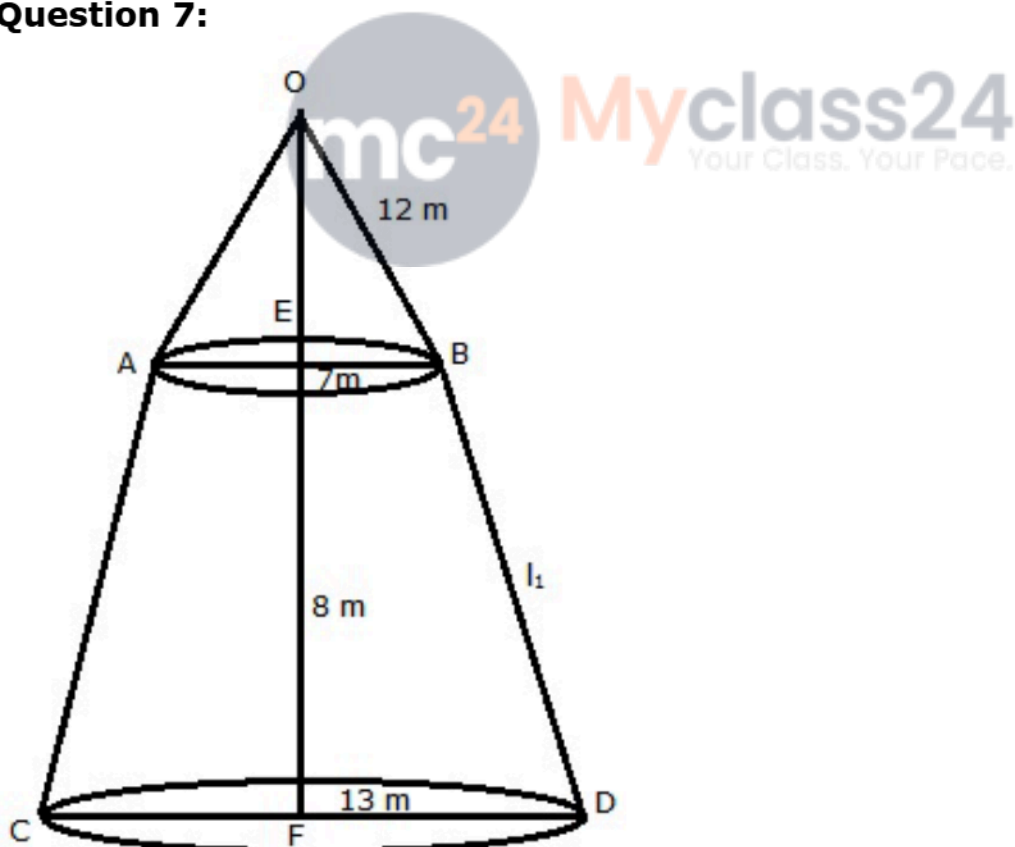
and $h = 4 \text{ m}$

$$\begin{aligned}
 \therefore l_1 &= \sqrt{3^2 + 4^2} \text{ m} \\
 &= \sqrt{25} \text{ m} = 5 \text{ m}
 \end{aligned}$$

Quantity of canvas = (Lateral surface area of the frustum) + (lateral surface area of the cone)

$$\begin{aligned}
 &= [\pi l (R + r) + \pi r l_1] \text{ m}^2 \\
 &= \pi [25 \times (10 + 3) + (3 \times 5)] \text{ m}^2 \\
 &= \frac{22}{7} \times [(25 \times 13) + (3 \times 5)] \text{ m}^2 \\
 &= 1068.57 \text{ m}^2
 \end{aligned}$$

Question 7:



ABCD is the frustum in which upper and lower radii are $EB = 7 \text{ m}$ and $FD = 13 \text{ m}$

Height of frustum = 8 m

Slant height l_1 of frustum

$$\begin{aligned}
 &= \sqrt{h^2 + (R - r)^2} \\
 &= \sqrt{8^2 + (13 - 7)^2} \\
 &= \sqrt{64 + 36} \\
 &= \sqrt{100} = 10 \text{ m}
 \end{aligned}$$

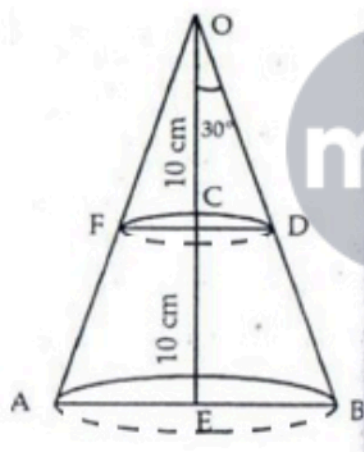
Radius of the cone = EB = 7 m

Slant height l_2 of cone = 12 m

Surface area of canvas required

$$\begin{aligned}
 &= \pi(R + r)l_1 + \pi r l_2 \\
 &= \pi[(13 + 7) \times 10 + 7 \times 12] \\
 &= \frac{22}{7} \times [200 + 84] = \frac{22}{7} \times 284 \text{ m}^2 \\
 &= 892.6 \text{ m}^2
 \end{aligned}$$

Question 8:



In the given figure, we have

$\angle COD = 30^\circ$, OC = 10 cm, OE = 20 cm

Let CD = r cm and EB = R cm



$$\frac{CD}{OC} = \tan 30^\circ$$

$$\Rightarrow \frac{CD}{10} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow CD = \left(10 \times \frac{1}{\sqrt{3}}\right) \text{ cm}$$
$$= \frac{10}{\sqrt{3}} \text{ cm}$$

$$\frac{EB}{OE} = \tan 30^\circ = \frac{EB}{20} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow EB = \left(20 \times \frac{1}{\sqrt{3}}\right) \text{ cm} \Rightarrow R = \frac{20}{\sqrt{3}} \text{ cm}$$

Also, $CE = 10 \text{ cm}$

Thus, $ABDF$ is the frustum of a cone in which

$$R = \frac{20}{\sqrt{3}} \text{ cm}, r = \frac{10}{\sqrt{3}} \text{ cm} \text{ and } h = 10 \text{ cm}$$

$$\begin{aligned} \text{Volume of frustum} &= \frac{1}{3} \pi h (R^2 + r^2 + Rr) \\ &= \frac{1}{3} \times \pi \times 10 \times \left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right) \\ &= \left(\frac{\pi \times 10}{3} \times \frac{700}{3}\right) \text{ cm}^3 = \left(\frac{7000\pi}{9}\right) \text{ cm}^3 \end{aligned}$$

Volume of wire of radius r and length l

$$= \pi r^2 l = \pi \left[\frac{1}{32}\right]^2 l$$

Volume of wire = Volume of frustum

$$\pi \left(\frac{1}{32}\right)^2 l = \frac{7000\pi}{9}$$

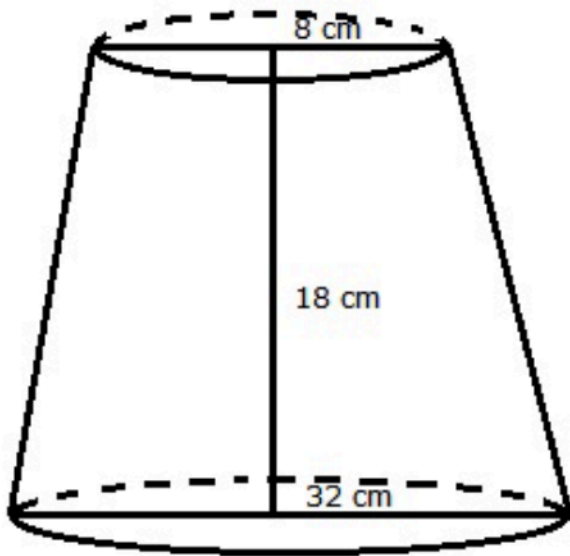
$$l = \frac{7000 \times 32 \times 32}{9} \text{ cm}$$

$$= \frac{70 \times 32 \times 32}{9} \text{ m}$$

$$= 7964.44 \text{ m}$$

Length of the wire is 7964.44 m

Question 9:



Radii of upper and lower end of frustum are $r = 8$ cm, $R = 32$ cm
Height of frustum $h = 18$ cm

$$\begin{aligned}\text{Volume of frustum} &= \frac{1}{3} \pi h [R^2 + r^2 + R \times r] \\ &= \frac{1}{3} \times \frac{22}{7} \times 18 \times [32^2 + 8^2 + 32 \times 8] \text{ cm}^3 \\ &= \frac{22 \times 6}{7} [1024 + 64 + 256] \text{ cm}^3 \\ &= \frac{132}{7} \times 1344 \text{ cm}^3 = 25344 \text{ cm}^3 = 25.344 \text{ litres}\end{aligned}$$

Cost of milk at Rs 20 per litre = Rs. $25.344 \times 20 =$ Rs. 506.88