NCERT Solutions for Class-XI Maths

Chapter-9 Exercise-9.3

NCERT Math Class 11

1. Find the 20^{th} and n^{th} terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

1. The given G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

Here, $a =$ First term $= \frac{5}{2}$

$r =$ Common ratio $= \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$

$$a_{20} = ar^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{(2)(2)^{19}} = \frac{5}{(2)^{20}}$$

$$a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{(2)(2)^{n-1}} = \frac{5}{(2)^n}$$

2. Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2.

2. Given: 8^{th} of the G.P. is 192 and common ratio is 2.

That is,

$a_8 = 192$ and $r = 2$.

We know that in G.P $a_n = ar^{n-1}$

Here, n: number of terms

a: First term = 5/2

r: common ratio

$\therefore a_8 = a(2)^{8-1}$ ($\because r = 2$ and $n = 8$)

$\Rightarrow 192 = a(2)^7$

$\Rightarrow a \times 128 = 192$

$\Rightarrow a = \frac{192}{128} = \frac{3}{2}$

Now,

$$a_{12} = \frac{3}{2} \times 2^{12-1} = \frac{3}{2} \times 2^{11} = 3 \times 2^{10} = 3072$$

$\therefore a_{12} = 3072$

3. The $5^{th}, 8^{th}$ and 11^{th} terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.

3. Let a be the first term and r be the common ratio of the G.P. According to the given condition,

$a_5 = ar^{5-1} = ar^4 = p$ …(1)

$a_8 = ar^{8-1} = ar^7 = q$ … (2)

$a_{11} = ar^{11-1} = ar^{10} = s$ … (3)

Dividing equation (2) by (1), we obtain

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$

$$r^3 = \frac{q}{p} \quad \dots (4)$$

Dividing equation (3) by (2), we obtain

$$\frac{ar^{10}}{ar^7} = \frac{s}{q}$$

$$\Rightarrow r^3 = \frac{s}{q} \quad \dots (5)$$

Equating the values of r^3 obtained in (4) and (5), we obtain

$$\frac{q}{p} = \frac{s}{q}$$

$$\Rightarrow q^2 = ps$$

Thus, the given result is proved.

4. The 4^{th} term of a G.P. is square of its second term, and the first term is – 3. Determine its 7^{th} term.

4. Given: $a_4 = (a_2)^2$ and $a = -3$

We know that in G.P $a_n = ar^{n-1}$

$\therefore a_2 = (-3) \times r^{2-1} = -3r \ (\because a = -3)$

Similarly,

$a_4 = -3r^3$

$\therefore -3r^3 = (-3r)^2 \ (\because a_4 = (a_2)^2)$

$\therefore -3r^3 = 9r^2$

$\Rightarrow r = -3$

$\therefore r = -3$

Now,

$a_7 = ar^{7-1}$

$\Rightarrow a_7 = (-3)(-3)^6$ ($\because a = -3$ and $r = -3$)

$\Rightarrow a_7 = (-3)^7 = -2187.$

$\therefore a_7 = -2187$

5. Which term of the following sequences:

(a) $2, 2\sqrt{2}, 4, \ldots$ is 128 ?

(b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ is 729 ?

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ is $\frac{1}{19683}$?

5. (a) The given sequence is $2, 2\sqrt{2}, 4, \ldots$ is 128 ?

Here, $a = 2$ and $r = \left(2\sqrt{2}\right)/2 = \sqrt{2}$

Let the n^{th} term of the given sequence be 128 .

$a_n = ar^{n-1}$

$\Rightarrow (2)(\sqrt{2})^{n-1} = 128$

$\Rightarrow (2)(2)^{\frac{n-1}{2}} = (2)^7$

$\Rightarrow (2)^{\frac{n-1}{2}+1} = (2)^7$

$\therefore \frac{n-1}{2} + 1 = 7$

$\Rightarrow \frac{n-1}{2} = 6$

$\Rightarrow n - 1 = 12$

$\Rightarrow n = 13$

Thus, the 13^{th} term of the given sequence is 128 .

(b) The given sequence is $\sqrt{3}, 3, 3\sqrt{3}, \ldots$

$a = \sqrt{3}$ and $r = \frac{3}{\sqrt{3}} = \sqrt{3}$

Let the n^{th} term of the given sequence be 729 .

$a_n = ar^{n-1}$

$\therefore ar^{n-1} = 729$

$\Rightarrow \left(\sqrt{3}\right)(\sqrt{3})^{n-1} = 729$

$\Rightarrow (3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} = (3)^6$

$\Rightarrow (3)^{\frac{1}{2}+\frac{n-1}{2}} = (3)^6$

$\therefore \frac{1}{2}+\frac{n-1}{2}=6$

$\Rightarrow \frac{1+n-1}{2}=6$

$\Rightarrow n=12$

Thus, the 12^{th} term of the given sequence is 729 .

(c) The given sequence is $\frac{1}{3},\frac{1}{9},\frac{1}{27},...$

Here, $a=\frac{1}{3}$ and $r=\frac{1}{9}\div\frac{1}{3}=\frac{1}{3}$

Let the n^{th} term of the given sequence be $\frac{1}{19683}$.

$a_n=ar^{n-1}$

$\therefore ar^{n-1}=\frac{1}{19683}$

$\Rightarrow \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1}=\frac{1}{19683}$

$\Rightarrow \left(\frac{1}{3}\right)^{n}=\left(\frac{1}{3}\right)^{9}$

$\Rightarrow n=9$

Thus, the 9^{th} term of the given sequence is $\frac{1}{19683}$.

6. For what values of x, the numbers $-\frac{2}{7},x,-\frac{7}{2}$ are in G.P.?

6. Given: $\frac{-2}{7}$, x, $\frac{-7}{2}$

For the given Sequence to be in G.P. Common ratio between two adjacent numbers in the sequence should be equal.

That is: Common ratio between $\frac{-2}{7}$, x = Common ratio between x, $\frac{-7}{2}$

$\therefore \frac{\frac{-2}{7}}{x}=\frac{x}{\frac{2}{7}}$

$\Rightarrow \left(\frac{-2}{7}\right)\left(\frac{-7}{2}\right)=(x)\times(x)$

$\Rightarrow (1)^2 = x^2$

$\Rightarrow x = \sqrt{(1)^2}$

$\Rightarrow x = \pm 1$

∴ For the given sequence to be in G.P. x should be ± 1.

7. Find the sum to 20 terms in the geometric progression $0.15, 0.015, 0.0015\ldots$

7. The given G.P. is $0.15, 0.015, 0.00015\ldots$

Here, $a = 0.15$ and $r = \frac{0.015}{0.15} = 0.1$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\therefore S_{20} = \frac{0.15\left[1-(0.1)^{20}\right]}{1-0.1}$$

$$= \frac{0.15}{0.9}\left[1-(0.1)^{20}\right]$$

$$= \frac{15}{90}\left[1-(0.1)^{20}\right]$$

$$= \frac{1}{6}\left[1-(0.1)^{20}\right]$$

8. $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots n$ terms.

8. Given: $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots$ n terms.

Sum of n terms of a G.P. is given by: $\frac{a\left(1-r^n\right)}{1-r}$ (a: First term of G.P, r: common difference of G.P, n: Number of terms of the G.P)

First term of the Given G.P (a) $= \sqrt{7}$

Common difference of the given G.P(r) $= \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$

Number of terms(n): n

Let the sum of n terms be s

$$\therefore s = \frac{a\left(1-r^n\right)}{1-r}$$

$$\Rightarrow s = \frac{\sqrt{7}\times\left(1-\left(\sqrt{3}\right)^n\right)}{1-\sqrt{3}}$$

$$\Rightarrow s = \frac{\sqrt{7}}{1-\sqrt{3}}\left[1-\left(\sqrt{3}\right)^n\right]$$

∴ The sum of n terms of the given sequence is: $\frac{\sqrt{7}}{1-\sqrt{3}}\left[1-\left(\sqrt{3}\right)^n\right]$

9. Find the sum to n terms in the geometric progression

$1,-a,a^2,-a^3 \ldots$ (if $a \neq -1$)

9. The given G.P. is $1,-a,a^2,-a^3,$

Here, first term $= a_1 = 1$

Common ratio $= r = -a$

$$S_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

$$\therefore S_n = \frac{1\left[1-(-a)^n\right]}{1-(-a)} = \frac{\left[1-(-a)^n\right]}{1+a}$$

10. $x^3, x^5, x^7, \ldots$ n terms (if $x \neq \pm 1$).

10. Given: $x^3, x^5, x^7, \ldots$ n terms

Sum of n terms of a G.P. is given by: $\frac{a\left(1-r^n\right)}{1-r}$ (a: First term of G.P, r: common difference of G.P, n: Number of terms of the G.P)

First term of the Given G.P (a) $= x^3$

Common difference of the given G.P(r) $= \frac{x^5}{x^3} = x^2$

Number of terms(n): n

Let the sum of n terms be s

$$\therefore s = \frac{x^3 \times \left(1-\left(x^2\right)^n\right)}{1-\left(x^2\right)}$$

$$\Rightarrow s = \frac{x^3 \times \left(1-\left(x^2\right)^n\right)}{1-\left(x^2\right)}$$

$\therefore$ The sum of n terms of the given sequence is: $\frac{x^3 \times \left(1-\left(x^2\right)^n\right)}{1-\left(x^2\right)}$

11. Evaluate $\sum_{k=1}^{11}\left(2+3^k\right)$

11. $$\sum_{k=1}^{11}\left(2+3^k\right) = \sum_{k=1}^{11}(2) + \sum_{k=1}^{11}3^k = 2(11) + \sum_{k=1}^{11}3^k = 22 + \sum_{k=1}^{11}3^k$$

$$\sum_{k=1}^{11}3^k = 3^1 + 3^2 + 3^3 + \ldots + 3^{11}$$

The terms of this sequence $3,3^2,3^3 \ldots$ forms a G.P.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_{11} = \frac{3\left[(3)^{11} - 1\right]}{3 - 1}$$

$$\Rightarrow S_{11} = \frac{3}{2}\left(3^{11} - 1\right)$$

$$\therefore \sum_{k=1}^{11} 3^k = \frac{3}{2}\left(3^{11} - 1\right)$$

Substituting this value in equation (1), we obtain

$$\sum_{k=1}^{11}\left(2 + 3^k\right) = 22 + \frac{3}{2}\left(3^{11} - 1\right)$$

12. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

12. Given: Sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1

Let $\frac{a}{r}$, a , r be the three terms in G.P.

$\therefore \frac{a}{r} + a + r = \quad -1$

and $\left(\frac{a}{r} \times a \times r\right) = 1 - 2$

$\Rightarrow a^3 = 1$

$\Rightarrow a = 1$

Substitute ‘a’ in –1, we get

$\frac{1}{r} + 1 + r = \frac{39}{10}$

$\Rightarrow 10 \times (1 + r + r^2) = 39 \times r$

$\Rightarrow 10 + 10 \times r + 10 \times r^2 = 39 \times r$

$\Rightarrow 10r^2 + 10r - 39r + 10 = 0$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$

$\Rightarrow (5r-2)(2r-5) = 0$

$\Rightarrow r = \frac{2}{5}, \frac{5}{2}$

$\therefore$ The three terms in the G.P. are $\frac{2}{5}, 1, \frac{5}{2}$

13. How many terms of G.P. $3, 3^2, 3^3 \ldots$ are needed to give the sum 120?

13. The given G.P. is $3, 3^2, 3^3 \ldots$

Let n terms of this G.P. be required to obtain the sum as 120 .

$$S_n = \frac{a(1-r^n)}{1-r}$$

Here, $a = 3$ and $r = 3$

$$\therefore S_n = 120 = \frac{3(3^n - 1)}{3-1}$$

$$\Rightarrow 120 = \frac{3(3^n - 1)}{2}$$

$$\Rightarrow \frac{120 \times 2}{3} = 3^n - 1$$

$$\Rightarrow 3^n - 1 = 80$$

$$\Rightarrow 3^n = 81$$

$$\Rightarrow 3^n = 3^4$$

$$\therefore n = 4$$

14. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

14. Given: The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128.

Let a, ar, ar^2, ar^3, ar^4, ar^5 be the terms of the G.P

Here $a + ar + ar^2 = 16$ (∵ given that sum of first 3 terms is 16) --------–1

Also, ar^3, ar^4, $ar^5 = 128$ (∵ given that sum of next 3 terms is 128)---------------–2

Divide eq –2 and eq –1

We get,

$$\frac{ar^3 + ar^4 + ar^5}{(a + ar + ar^2)} = \frac{128}{16}$$

$$\Rightarrow \frac{ar^3 \times (1 + r + r^2)}{a \times (1 + r + r^2)} = 8$$

$\Rightarrow r^3 = 8$

$\Rightarrow r = \sqrt[3]{8}$

$\Rightarrow r = 2$

Here $a + ar + ar^2 = 16$

$\Rightarrow a \times (1 + r + r^2) = 16$

$\Rightarrow a \times (1 + 2 + (2)^2) = 16$

$\Rightarrow a \times (1 + 2 + 4) = 16$

$\Rightarrow a \times (7) = 16$

$\Rightarrow a = \frac{16}{7}$

The Sum of terms in G.P. is given by: $\frac{a\left(r^n - 1\right)}{r - 1}$

$\therefore \frac{a\left(r^n - 1\right)}{r - 1} = \frac{\frac{16}{7} \times \left(2^n - 1\right)}{2 - 1} = \frac{16 \times \left(2^n - 1\right)}{7}$

$\therefore$ First term of the G.P is $\frac{16}{7}$, common ratio of the G.P is 2 and Sum of n terms of the G.P is $\frac{16}{7}\left(2^n - 1\right)$

15. Given a G.P. with $a = 729$ and 7^{th} term 64 , determine S7.

15. $a = 729 a_7 = 64$

Let r be the common ratio of the G.P. It is known that,

$a_n = ar^{n-1}$

$a_7 = ar^{7-1} = (729)r^6$

$\Rightarrow 64 = 729r^6$

$\Rightarrow r^6 = \frac{64}{729}$

$\Rightarrow r^6 = \left(\frac{2}{3}\right)^6$

$\Rightarrow r = \frac{2}{3}$

Also, it is known that,

$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$

$\therefore S_7 = \frac{729\left[1 - \left(\frac{2}{3}\right)^7\right]}{1 - \frac{2}{3}}$

$= 3 \times 729\left[1 - \left(\frac{2}{3}\right)^7\right]$

$=(3)^7\left[\frac{(3)^7-(2)^7}{(3)^7}\right]$

$=(3)^7-(2)^7$

$=2187-128$

$=2059$

16. Find a G.P. for which sum of the first two terms is – 4 and the fifth term is 4 times the third term.

16. Given: $S_2 = -4$ and $a_5 = 4 \times a_3$

The Sum of terms in G.P. is given by: $\frac{a\left(1-r^n\right)}{1-r}$

$\therefore S_2 = \frac{a\left(1-r^2\right)}{1-r} = -4$

$\Rightarrow \frac{a(1-r)(1+r)}{(1-r)} = -4$

$\Rightarrow a(1+r) = -4$ –1

Here,

$a_5 = 4 \times a_3$

$\Rightarrow ar^4 = 4 \times ar^2$ ($\because$ nth term of the G.P is ar^{n-1})

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

$\therefore r = +2$ or -2

Case 1: $r = +2$

From eq –1

$a(1+r) = -4$

$\Rightarrow a(1+2) = -4$

$\Rightarrow a = \frac{-4}{3}$

$\therefore$ G.P : $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots\ldots$

Case 2: $r = -2$

From eq –1

$a(1+r) = -4$

$\Rightarrow a(1+(-2)) = -4$

$\Rightarrow a = \frac{-4}{-1} = 4$

$\therefore$ G.P : 4 , –8, 16, –32,…….

$\therefore$ The possible G.P 's are $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots\ldots$ or 4 , –8, 16, –32,…….

17. If the $4^{th}, 10^{th}$ and 16^{th} terms of a G.P. are x, y and z, respectively. Prove that x, y, z are in G.P.

17. Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$a_4 = ar^3 = x.$

$a_{10} = ar^9 = y$

$a_{16} = ar^{15} = z$

Dividing (2) by (1), we obtain

$\frac{y}{x} = \frac{ar^9}{ar^3} \Rightarrow \frac{y}{x} = r^6$

Dividing (3) by (2), we obtain

$\frac{z}{y} = \frac{ar^{15}}{ar^9} \Rightarrow \frac{z}{y} = r^6$

$\frac{y}{x} = \frac{z}{y}$

Thus, x, y, z are in G. P.

18. Find the sum to n terms of the sequence 8, 88, 888, 8888… .

18. Given: sequence: 8, 88, 888, 8888… .

$S_n = 8 + 88 + 888 + 8888 + \ldots\ldots.$

$\Rightarrow S_n = 8[1 + 11 + 111 + 1111 + \ldots\ldots.]$

$\Rightarrow S_n = \frac{8}{9} \times [9 + 99 + 999 + 9999 + \ldots\ldots.]$

$\Rightarrow S_n = \frac{8}{9} \times [(10 - 1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \ldots\ldots.]$

$\Rightarrow S_n = \frac{8}{9} \times [10 + 10^2 + 10^3 + 10^4 + \ldots\ldots.] - \frac{8}{9} \times [1 + 1 + 1 + 1 + \ldots\ldots.]$

$\Rightarrow S_n = \frac{8}{9} \times [10 + 10^2 + 10^3 + 10^4 + \ldots\ldots.] - \frac{8}{9} \times [1 + 1 + 1 + 1 + \ldots\ldots.]$

$\Rightarrow S_n = \frac{8}{9} \times \left[\frac{a(r^n - 1)}{r - 1}\right] - \frac{8}{9} \times (n)$ ($\because$ sum of n terms of G.P is $\frac{a(r^n - 1)}{r - 1}$, here $a = 10$ and $r = 10$)

$\Rightarrow S_n = \frac{8}{9} \times \left[\frac{10 \times (10^n - 1)}{10 - 1}\right] - \frac{8}{9} \times (n)$

$$\Rightarrow S_n = \frac{8}{9} \times \left[\frac{10 \times \left(10^n - 1\right)}{9}\right] - \frac{8}{9} \times (n)$$

$$\Rightarrow S_n = \frac{80}{81} \times \left[10^n - 1\right] - \frac{8}{9} \times (n)$$

$$\therefore S_n = \frac{80}{81} \times \left[(10^n - 1\right] - \frac{8}{9} \times (n)$$

19. Find the sum of the products of the corresponding terms of the sequences $2,4,8,16,32$ and $128,32,8,2,1/2$.

19. Required sum

$$= 2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2}$$

$$= 64\left[4 + 2 + 1 + \frac{1}{2} + \frac{1}{2^2}\right]$$

Here, $4,2,1,\frac{1}{2},\frac{1}{2^2}$ is a G.P.

First term, $a = 4$

20. Show that the products of the corresponding terms of the sequences a, ar, ar^2,…ar^{n-1} and A, AR, AR^2, … AR^{n-1} form a G.P, and find the common ratio.

20. Given: The sequences a, ar, ar^2,…ar^{n-1} and A, AR, AR^2, … AR^{n-1}

The products of the corresponding terms of the G.P is

$a \times A$, $ar \times AR$, $ar^2 \times AR^2$, ……….., $ar^n \times AR^n$

Here,

$$\frac{\text{second term}}{\text{first term}} = \frac{ar \times AR}{a \times A} = rR$$

$$\frac{\text{third term}}{\text{second term}} = \frac{ar^2 \times AR^2}{ar \times AR} = rR$$

$\therefore$ The product of the corresponding terms of the given sequence forms a G.P and the common ratio is rR

21. Find four numbers forming a geometric progression in which third term is greater than the first term by 9 , and the second term is greater than the 4^{th} by 18 .

21. Let a be the first term and r be the common ratio of the G.P.

$$a_1 = a,\ a_2 = ar,\ a_3 = ar^2,\ a_4 = ar^3$$

By the given condition,

$$a_3 = a_1 + 9 \Rightarrow ar^2 = a + 9 \qquad \ldots(1)$$

$a_2 = a_4 + 18 \Rightarrow ar = ar^3 + 18$ …(2)

From (1) and (2), we obtain

$a(r^2 - 1) = 9$ …(3)

$ar(1 - r^2) = 18$ …(4)

Dividing (4) by (3), we obtain

$$\frac{ar(1-r^2)}{a(r^2-1)} = \frac{18}{9}$$

$\Rightarrow -r = 2$

$\Rightarrow r = -2$

Substituting the value of r in (1), we obtain

$4a = a + 9$

$\Rightarrow 3a = 9$

$\therefore a = 3$

Thus, the first four numbers of the G.P. are $3, 3(-2), 3(-2)^2$, and $3(-2)^3$ i.e., $3, -6, 12$, and -24.

22. If the p^{th}, q^{th} and r^{th} terms of a G.P are a, b and c, respectively. Prove that $a^{q-r} \times b^{r-p} \times c^{P-q} = 1$.

22. Given p^{th}, q^{th} and r^{th} terms of a G.P are a, b and c, respectively

Here

$a_p = a = ar^{p-1}$

$a_q = b = ar^{q-1}$

$a_r = c = ar^{r-1}$

Now,

$a^{q-r} \times b^{r-p} \times c^{P-q} = (ar^{p-1})^{q-r} \times (ar^{q-1})^{r-p} \times (ar^{r-1})^{p-q}$

$\Rightarrow a^{q-r} \times b^{r-p} \times c^{P-q} = (a^{(q-r)} \times r^{(p-1)(q-r)}) \times (a^{(r-p)} \times r^{(q-1)(r-p)}) \times (a^{(p-q)} \times r^{(r-1)(p-q)})$

$\Rightarrow a^{q-r} \times b^{r-p} \times c^{P-q} = (a^{(q-r)} \times r^{(pq-q-pr+r)}) \times (a^{(r-p)} \times r^{(qr-r-pq+p)}) \times (a^{(p-q)} \times r^{(pr-p-qr+q)})$

$\Rightarrow a^{q-r} \times b^{r-p} \times c^{P-q} = (a^{(q-r+r-p+p-q)} \times r^{(pq-q-pr+r+qr-r-pq+p+pr-p-qr+q)})$

$\Rightarrow a^{q-r} \times b^{r-p} \times c^{P-q} = (a^0 \times r^0) = 1$

$\therefore a^{q-r} \times b^{r-p} \times c^{P-q} = 1$

Hence proved.

23. If the first and the n^{th} term of a G.P. are a ad b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

23. The first term of the G.P is a and the last term is b.

Therefore, the G.P. is $a, ar, ar^2, ar^3 \ldots ar^{n-1}$, where r is the common ratio.

$b = ar^{n-1}$ ...(1)

$P =$ Product of n terms

$= (a)(ar)(ar^2)\ldots(ar^{n-1})$

$= (a \times a \times \ldots a)(r \times r^2 \times \ldots r^{n-1})$

$= anr1 + 2 + \ldots(n-1)$ ...(2)

Here, $1, 2, \ldots (n-1)$ is an A.P.

$\therefore 1 + 2 + \ldots\ldots\ldots + (n-1)$

$$= \frac{n-1}{2}\left[2 + (n-1-1) \times 1\right] = \frac{n-1}{2}[2 + n - 2] = \frac{n(n-1)}{2}$$

$P = a^n r^{\frac{n(n-1)}{2}}$

$\therefore P^2 = a^{2n} r^{n(n-1)}$

$= \left[a^2 r^{(n-1)}\right]^n$

$= \left[a \times ar^{n-1}\right]^n$

$= (ab)^n$ [Using(1)]

Thus, the given result is proved.

24. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$

24. Let *a* be the first term and *r* be the common ratio of the G.P.

Sum of n terms $= \frac{a(1-r^n)}{(1-r)}$

Since there are *n* terms from (*n* +1)th to (2*n*)th term,

Sum of terms from(*n* + 1)th to (2*n*)th term $= \frac{a_{n+1}(1-r^n)}{(1-r)}$

Here,

$a_{n+1} = ar^{n+1-1} = ar^n$

$\therefore$ Required ratio $= \frac{a(1-r^n)}{(1-r)} \times \frac{(1-r)}{ar^n(1-r^n)}$

∴ The ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)$th to $(2n)$th term is $= \frac{1}{r^n}$.

25. If a, b, c and d are in G.P. show that:

$$\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=(ab+bc-cd)^2$$

25. a, b, c, d are in G.P. Therefore,

$$bc = ad$$

$$b^2 = ac$$

$$c^2 = bd$$

It has to be proved that,

$$\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=(ab+bc-cd)^2$$

R.H.S.

$$=(ab+bc+cd)^2$$

$$=(ab+ad+cd)^2$$

[Using (1)]

$$=[ab+d(a+c)]^2$$

$$=a^2b^2+2abd(a+c)+d^2(a+c)^2$$

$$=a^2b^2+2a^2bd+2acbd+d^2\left(a^2+2ac+c^2\right)$$

$$=a^2b^2+2a^2c^2+2b^2c^2+d^2a^2+2d^2b^2+d^2c^2 \text{ [Using (1) and (2)]}$$

$$=a^2b^2+a^2c^2+a^2c^2+b^2c^2+b^2c^2+d^2a^2+d^2b^2+d^2b^2+d^2c^2$$

$$=a^2b^2+a^2c^2+a^2d^2+b^2\times b^2+b^2c^2+b^2d^2+c^2b^2+c^2\times c^2+c^2d^2$$

[Using (2) and (3) and rearranging terms]

$$=a^2\left(b^2+c^2+d^2\right)+b^2\left(b^2+c^2+d^2\right)+c^2\left(b^2+c^2+d^2\right)$$

$$=\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)= \text{ L.H.S.}$$

∴ L.H.S. = R.H.S.

$\therefore\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=(ab+bc-cd)^2$

26. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

26. Let a_1 and a_2 be two numbers between 3 and 81 such that the series, 3, a_1, a_2, 81, forms a G.P.

Let a_0 be the first term and r be the common ratio of the G.P.

$\therefore 81 = ar^3$

$\Rightarrow 81 = (3)r^3$

$\Rightarrow r^3 = 27$

$\therefore r = 3$

$a_1 = a_0 r = (3)(3) = 9$

$a_2 = a_0 r^2 = (3)(3)^2 = 27$

$\therefore$ The required two numbers are 9 and 27.

27. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

27. M. of a and b is $\sqrt{ab}$

By the given condition: $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\sqrt{ab}$

Squaring both sides, we obtain

$$\frac{\left(a^{n+1}+b^{n+1}\right)^2}{\left(a^n+b^n\right)^2}=ab$$

$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=(ab)\left(a^{2n}+2a^nb^n+b^{2n}\right)$

$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=a^{2n+1}b+2a^{n+1}b^{n+1}+ab^{2n+1}$

$\Rightarrow a^{2n+2}+b^{2n+2}=a^{2n+1}b+ab^{2n+1}$

$\Rightarrow a^{2n+2}-a^{2n+1}b=ab^{2n+1}-b^{2n+2}$

$\Rightarrow a^{2n+1}(a-b)=b^{2n+1}(a-b)$

$\Rightarrow \left(\frac{a}{b}\right)^{2n+1}=1=\left(\frac{a}{b}\right)^0$

$\Rightarrow 2n+1=0$

$\Rightarrow n=\frac{-1}{2}$

28. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

28. Let the two numbers be a and b.

G.M. = $\sqrt{ab}$

According to the given condition,

$a + b = 6\sqrt{ab}$

squaring on both sides we get,

$\Rightarrow (a + b)^2 = 36(ab)$ …(1)

Here,

$(a—b)^2 = (a + b)^2 – 4ab = 36ab – 4ab = 32ab$

$\Rightarrow a—b = \sqrt{32}\sqrt{ab} = 4\sqrt{2}\sqrt{ab}$ …(2)

Adding eq(1) and eq(2), we get

$2a = (6 + 4\sqrt{2})\sqrt{ab}$

$\Rightarrow a = (3 + 2\sqrt{2})\sqrt{ab}$

Substituting the value of *a* in eq(1), we obtain

$b = 6\sqrt{ab} – (3 + 2\sqrt{2})\sqrt{ab}$

$\Rightarrow b = (3 — 2\sqrt{2})\sqrt{ab}$

Now,

$$\frac{a}{b} = \frac{(3+2\sqrt{2})\sqrt{ab}}{(3-2\sqrt{2})\sqrt{ab}} = \frac{(3+2\sqrt{2})}{(3-2\sqrt{2})}$$

$\therefore$ The required ratio is $(3+2\sqrt{2}):(3-2\sqrt{2})$

29. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$

29. It is given that A and G are A.M. and G.M. between two positive numbers.

Let these two positive numbers be a and b.

$\therefore AM = A = \frac{a+b}{2}$ ...(1)

$GM = G = \sqrt{ab}$ ...(2)

From (1) and (2), we obtain

$a + b = 2A$ …(3)

$ab = G^2$ …(4)

Substituting the value of a and b from (3) and (4) in the identity

$(a-b)^2 = (a+b)^2 - 4ab$

we obtain

$(a-b)^2 = 4A^2 - 4G^2 = 4(A^2 - G^2)$

$(a-b)^2 = 4(A+G)(A-G)$

$(a-b) = 2\sqrt{(A+G)(A-G)}$

From (3) and (5), we obtain

$2a = 2A + 2\sqrt{(A+G)(A-G)}$

$\Rightarrow \mathbf{a} = A + \sqrt{(A+G)(A-G)}$

Substituting the value of a in (3), we obtain

$b = 2A - A - \sqrt{(A+G)(A-G)} = A - \sqrt{(A+G)(A-G)}$

Thus, the two numbers are $A \pm \sqrt{(A+G)(A-G)}$

30. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and nth hour?

30. Given: there were 30 bacteria present originally and culture doubles every hour.

$\therefore a_0 = 30$

$\therefore$ After an hour culture is $a_0 \times 2 = 30 \times 2 = 60$

Since the count doubles every hour it forms a G.P. with r = 2

Let the G.P be a_0, a_1r, a_2r^2........

$\Rightarrow$ The culture at the end of 2^{nd} hour = $ar^2 = 30 \times 2^2 = 120$

$\Rightarrow$ The culture at the end of 4^{th} hour = $ar^4 = 30 \times 2^4 = 480$

$\Rightarrow$ The culture at the end of n^{th} hour = $ar^n = 30 \times 2^n$

$\therefore$ Culture at end pf 2^{nd}, 4^{th}, n^{th} hours are 120, 480, 30×2^n respectively.

31. What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

31. The amount deposited in the bank is Rs 500 .

At the end of first year, amount $= Rs500\left(1+\frac{1}{10}\right) = Rs500(1.1)$

At the end of 2^{nd} year, amount $=$ Rs 500 (1.1) (1.1)

At the end of 3^{rd} year, amount $=$ Rs 500 (1.1) (1.1) (1.1) and so on

:Amount at the end of 10 years $=$ Rs 500 (1.1) (1.1) ... (10 times)

$=$ Rs $500(1.1)^{10}$

32. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

32. Given: A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively.

Let a and b be the roots of the quadratic equation.

Here,

$\text{A.M} = \frac{a+b}{2} = 8$

$\Rightarrow a+b = 16$ …(1)

$\text{G.M} = \sqrt{ab} = 5$

$\Rightarrow (\sqrt{ab})^2 = 5^2$

$\Rightarrow ab = 25$ …(2)

The quadratic equation is given by,

$x^2 - x$ (Sum of roots) + (Product of roots) = 0

$x^2 - x(a+b) + (ab) = 0$

$x^2 - 16x + 25 = 0$ (From eq –1 and eq –2)

∴ The required quadratic equation is $x^2 - 16x + 25 = 0$